JENA ECONOMIC RESEARCH PAPERS

2012 – 014

Persuasive Silence

by

Toru Suzuki

www.jenecon.de

ISSN 1864-7057

The JENA ECONOMIC RESEARCH PAPERS is a joint publication of the Friedrich Schiller University and the Max Planck Institute of Economics, Jena, Germany. For editorial correspondence please contact markus.pasche@uni-jena.de.

Impressum:

Friedrich Schiller University Jena
Carl-Zeiss-Str. 3
D-07743 Jena
www.uni-jena.de

Max Planck Institute of Economics
Kahlaische Str. 10
D-07745 Jena
www.econ.mpg.de

Jena Economic Research Papers 2012 - 014

Persuasive Silence *

Toru Suzuki†

March 29, 2012

Abstract

In the market where inattentive buyers can fail to notice some feasible choices, the key role of marketing is to make buyers aware of products. However, the effective marketing strategy is often subtle since marketing tactics can make buyers cautious. This paper provides a framework to analyze an effective marketing strategy to persuade an inattentive buyer in an adverse selection environment. We investigate how an attention-grabbing marketing can "backfire" and when it can be effective.

Keywords. Signaling game, Consideration set, Counter signaling, Limited attention, Marketing, Advertising

JEL Code: D03, D82, D83, L15

*I thank Jacob Glazer, Hsueh-Ling Huynh, Barton Lipman, John Wooders, Andriy Zapechelnyuk and seminar participants in IUPUI, Keio University, Monash University, Queen Mary University of London, Syracuse University, University of Technology Sydney for helpful comments.

†Max Planck Institute of Economics, Jena, Germany

"Silence is golden, speech is silver" - Thomas Carlyle

"Don't you know that silence supports the accuser's charge?"- Sophocles

1 Introduction

A decision maker with limited attention can fail to notice the existence of some feasible choices. Thus, one of the main purposes of marketing and other self promotion activities is to make decision makers aware of a feasible choice. For example, firms try to get consumers' attention by advertising. Job candidates contact their potential employers to promote themselves.

Even though such an "attention-grabbing marketing" often works, the effective marketing strategy can be more subtle in other situations. Consider a situation where a salesperson visits a potential customer's house to sell his product. Even if such a marketing strategy can make the potential customer aware of his product, this strategy often makes the consumer cautious and discourages her from buying it, that is, an attention-grabbing marketing can backfire. This paper provides a simple framework to investigate how an attention-grabbing marketing can backfire and when it can be effective.

If a buyer can fail to notice the existence of feasible choices, such an attention-grabbing marketing seems to be a natural strategy for the seller. Thus, the buyer's negative reaction to the attention-grabbing marketing is seemingly "irrational" response. However, our paper shows that when the buyer's search technology for products depends on the product quality, the buyer's negative reaction can be consistent with a rational reasoning. To see the idea, suppose that the buyer tends to get to know high quality products through her friends or neighbors, i.e., word-of-mouth. Then, the buyer might think that the seller needs to catch her attention because he knows the chance that she already knew his product is low. That is, the fact that he has to catch her attention reveals his product quality. A higher quality type is confident that she already knew his product and he does not try to catch her attention intentionally in order to distinguish himself from lower types. Hence, the buyer's negative response to the attention-grabbing marketing can be, indeed, consistent with the equilibrium belief in a signaling game.

In Section 2, we introduce the model. The game is a signaling game. There is a seller who wishes to sell a product to a buyer. The value of the product is the seller's private information. The buyer's attention is limited and can fail to notice some feasible choices. Then, the buyer chooses the best product from her consideration set which consists of products she is aware of. The formation of her consideration set depends on her search outcome and the seller's marketing strategy. Concretely, the buyer searches high quality products to construct her consideration set. The search technology is assumed to be stochastic and she finds the seller's product with higher probability if the product quality is higher. On the other hand, without knowing whether the buyer already noticed the seller's product, the seller decides whether to make her notice his product by sending a costless message, i.e., an attention-grabbing marketing. The buyer then chooses a product from her consideration set.

In Section 3, we analyze the equilibria and characterize them. Since the seller's payoff function is independent of his action and type, the game always has uninformative equilibria. Hence, our first question is whether any informative equilibrium can exist. It is shown that if the buyer's search technology can filter out low quality products with a high probability, an informative equilibrium exists. Furthermore, it is shown that any informative equilibrium is a countersignaling equilibrium, that is, lower types send a message, while the higher types remain silent.

As other signaling games, our game has multiple equilibria. In Section 4, we analyze which equilibrium is best for each player. First, on the contrary to Spencian signaling models, the receiver can prefer a pooling equilibrium to informative equilibria in our model. This situation occurs when the search technology often fails to find the seller's product for any quality and the seller's informative signal, i.e., remaining silent, "costs" too much for the buyer. On the other hand, if the search technology can sort out high quality products with a high probability, the buyer can prefer a countersignaling equilibrium to pooling equilibria. Thus, in this case, the buyer's "limitation of attention" makes the marketing strategy an effective signaling device and it can improve efficiency of the transaction in the adverse selection environment.

In Section 5, we analyze the refinement of the equilibrium set with perfect sequential equilibrium (PSE). Since there is no off-the-equilibrium action in informative equilibria, any informative equilibrium is a PSE. Then, our question is whether each pooling equilibrium is a PSE. First, we show that the pooling equilibrium in which all types remain silent is not a

PSE. Second, it is shown that if the buyer's search rarely misses a product with very high quality, any PSE can be a countersignaling equilibrium.

The rest of the paper proceeds as follows. In section 6, we extend the basic model. First, we show that the main result is preserved as long as the cost of messaging is relatively low given the payoff from selling. Second, we extend our model to a multi-seller setting. In section 7, based on our framework, we discuss situations in which an attention-grabbing marketing works without backfiring. Section 8 summarizes the paper.

Related literature. Our model is a signaling game in which the sender's payoff function is independent of the choice of a signal. Hence, our model is not a Spencian signaling model where the single crossing property of the payoff function makes the signal informative. On the other hand, our signaling game departs from cheap talk game in Crawford and Sobel (1982) in two aspects. First, our sender's payoff function is independent of the sender's type. Second, the sender's message can make the receiver aware of a feasible choice, that is, the message can affect the receiver's choice set in our model.

In our game, any informative equilibrium is a countersignaling equilibrium, that is, higher types remain silent, while lower types send a message. Feltovich et al (2002) analyzes a countersignaling equilibrium in a Spencian signaling model. The key ingredient of their model is an exogenous signal which statistically reflects the sender's type. When the exogenous signal is sufficiently accurate for the high type, he does not need to signal his type. On the other hand, the middle type, who cannot rely on the exogenous signal, sends a costly signal to distinguish himself from the low type. Our model also has an element which can be interpreted as an exogenous signal. Our receiver infers the quality not only from the sender's signaling but also her search outcome. Then, since the probability of finding the seller's product depends on the product quality, the search outcome is an exogenous signal of the product quality. On the other hand, even though an exogenous signal plays a key role in both models, the mechanics behind the countersignaling is different between two models. While the existence of informative equilibria relies on the single crossing property of the payoff function in Feltovich et al (2002), our informative equilibria can exist even though the sender's payoff function is independent of his signal and type. Moreover, unlike their model, any informative equilibrium is a countersignaling equilibrium in our model.

Our paper contributes to models of advertising in which advertising affects the purchasing decision through the buyer's belief, i.e., informative advertising. Stigler (1961) and Nelson (1974) explain the role of advertising as a provision of information about products such as their existence and price. Milgrom and Roberts (1986) analyzes advertising as a Spencian signaling. Our paper also contributes to the literature which analyzes how a seller can manipulate boundedly rational consumers. Rubinstein (1993) analyzes how a monopolist can get a higher profit from bounded rational consumers through a price setting strategy. Mullainathan et al. (2008) analyzes the role of salient messages in persuasion when the decision maker is a "coarse thinker." Shapiro (2006) develops a model in which advertising affects the buyer's recall process of the consumption experience. Our paper is closest to Eliaz and Spiegler (2011a, 2011b) which analyze how marketing can manipulate consumers' choices through their consideration sets.

Consideration set is an important concept in marketing. The basic idea is that the consumption decision follows a two-step decision process. Consumers first form a small set of options that they will consider for their consumption decision. Then, they evaluate the options in this set and choose the one they prefer the most. There are two types of papers on consideration sets in economics. The first type focuses on the individual choice problem to understand the property of consideration set procedures, e.g., Manzini and Mariotti (2007), Masatlioglu and Nakajima (2008), and Masatlioglu et al (2009). The second type analyzes economic interactions given a specific rule of the consideration set formation, i.e., Eliaz and Spiegler (2011a, 2011b). Our paper is categorized as the second type. In their seminal work, Eliaz and Spiegler analyze a complete information game where the consideration set is perfectly determined by the marketing strategy. Given a consideration set function, each seller chooses the marketing strategy to "stand out" in the market. On the other hand, our paper focuses on the situation where the product quality is the seller's private information, i.e., the product is an experience good in the sense of Nelson (1974). We assume that the consideration set consists of products the buyer is aware of and it is determined by the buyer's search outcome and the seller's marketing strategy. Given this rule of the consideration set formation, we analyze the effective marketing strategy which takes into account the signaling effect. Hence, in our model, the effective marketing strategy is not all about "standing out."

2 Model

There is a seller who wishes to sell product x_1 to a buyer. The value of x_1, denoted by θ, is the seller's private information and θ is drawn from distribution $F(\theta)$.[1] We assume that $F(\theta)$ has a continuous density function f with $supp(f) = \Theta = [\theta_{\min}, \theta_{\max}]$.

The buyer's attention is limited and she uses the following two step decision procedure. In the first step, she forms her consideration set X which consists of feasible products she is aware of.[2] In the second step, she chooses a product from her consideration set X. We assume that the buyer has two ways to notice a feasible product. The first way is a search. Here search means any activity whose aim is to find high quality products. For example, she may ask her friends or use the internet to search high quality products. Then, the buyer's **search technology** $\lambda(\theta)$ is defined as the probability that she finds product x_1 conditional on θ.[3] Since the buyer searches high quality products, we assume that the probability of finding x_1 is higher if the quality is higher.[4] Formally, we assume the following.

Assumption 1. *$\lambda(\theta)$ is continuous and strictly increasing in θ. Moreover, $\lambda(\theta) \in (0, 1)$ for any $\theta \in \Theta$.*

To interpret this assumption, consider a buyer who searches a high quality product. The assumption says that if she searches with her friends, her friends tend to mention x_1 with higher probability if the quality is higher, i.e., word-of-mouth. On the other hand, if she searches with the internet, she tends to encounter x_1 more frequently if the quality is higher.

For competing products of x_1 in X, we only specify the distribution of the best competing product. Formally, let x_0 be the best competing product of x_1 in X. Since the seller cannot observe products the buyer is aware of, the value of x_0, denoted by θ_0, is the buyer's private information and drawn from distribution $G(\theta_0)$. We assume that $G(\theta_0)$ has a continuous

[1] Since the buyer cannot evaluate the product quality θ in advance, product x_1 can be interpreted as an "experience good" in Nelson (1974).

[2] Shocker et al (1991) defines consideration set as a subset of "awareness set," that is, the decision maker may not evaluate some choices even if she is aware of them. Since the main interest of our paper is in how the buyer notices the existence of choices, we focus on the case in which the consideration set is the same as the awareness set.

[3] For simplicity, we assume the search is costless. In more general setting, the buyer can choose the search intensity given a cost function. In such a case, $\lambda(\theta)$ is interpreted as the search technology given the optimal search intensity.

[4] In Section 7, we discuss the case where this assumption does not hold.

density g with $\text{supp}(g) = \Theta_0 = [\theta_{\min}, \theta_{\max}]$. Note that the buyer always chooses either x_0 or x_1. Hence, without loss of generality, we assume that X is either $\{x_0\}$ or $\{x_0, x_1\}$.

The second way to notice product x_1 is receiving a message from the seller. Concretely, by sending a costless message, $a = M$, the seller can make the buyer aware of x_1.[5] In other words, the seller's message can ensure that the buyer is aware of his product. Here sending a message means any kind of action which can make the buyer aware of x_1. For instance, if the seller is a salesperson, sending a message can mean visiting the buyer's house. If the seller is a job candidate, sending a message may mean sending an E-mail to a potential employer. If the seller is a company, sending a message can mean advertising. On the other hand, if the seller remains silent, $a = N$, whether x_1 is in the buyer's consideration set or not only depends on her search outcome.

To clarify how her consideration set X is determined by the search and the seller's action, note that there are three possible scenarios for $X = \{x_0, x_1\}$. To see this, let $\omega \in \{0, 1\}$ be the buyer's search outcome, that is, $\omega = 1$ if the buyer found x_1 before the seller chooses action a, and $\omega = 0$ if the buyer could not find x_1 before the seller chooses a. First, when $\omega = 0$, $X = \{x_0, x_1\}$ only if the seller chooses $a = M$. Second, if $\omega = 1$, $X = \{x_0, x_1\}$ for both $a = M$ and N. Hence, $X = \{x_0, x_1\}$ if $(a, \omega) \in \{(M, 0), (N, 1), (M, 1)\}$. On the other hand, $X = \{x_0\}$ if $(a, \omega) = (N, 0)$. Thus, given the seller's action a and the value of x_1,

$$X = \begin{cases} \{x_0, x_1\} \text{ with probability } 1 & \text{if } a = M \\ \{x_0, x_1\} \text{ with probability } \lambda(\theta) & \text{if } a = N \\ \{x_0\} \text{ with probability } 1 - \lambda(\theta) & \text{if } a = N \end{cases}.$$

The payoff function of each player follows the standard adverse selection setting. The seller always wants the buyer to choose his product x_1. Concretely, if the buyer chooses x_1, the seller's payoff is 1 for any type, while his payoff is 0 if the buyer does not choose his product. On the other hand, the buyer's payoff depends only on the value of her choice, that is, if she chooses x_0, her payoff is θ_0, while her payoff is θ_1 if she chooses x_1. We assume that each player maximizes one's expected payoff.

Our game is a signaling game and the time line is as follows. First, the seller observes θ

[5]Meanwhile, we assume that sending a message is costless for simplicity. In Section 6, we introduce a cost to show the robustness of our result.

and the buyer searches feasible products. Then, the seller decides whether to send a message without knowing her search outcome ω. Formally, let $A = \{N, M\}$ be the set of the seller's actions. The seller's strategy is then a mapping[6]

$$\sigma : \Theta \to A.$$

After the seller chooses his action, the buyer chooses a product from her consideration set X. If $(a, \omega) \neq (N, 0)$ and $X = \{x_0, x_1\}$, the buyer infers θ conditional on (a, ω) and chooses one whose expected value is higher. On the other hand, if $(a, \omega) = (N, 0)$ and $X = \{x_0\}$, the buyer chooses x_0. Formally, let $H = \{(M, 1), (M, 0), (N, 1), \varnothing\}$ where $\varnothing$ denotes the case in which the buyer does not notice the existence of x_1. The buyer's strategy is then a mapping

$$s : H \times \Theta_0 \to X$$

such that $s(\varnothing, \theta_0) = x_0$ for any θ_0.

We employ **perfect Bayesian equilibrium** to analyze our game. Formally, (σ^*, s^*, μ^*) is **equilibrium** if

(i) for any θ, $\sigma^*(\theta)$ maximizes the seller's expected payoff given s^*.

(ii) for any (h, θ_0) such that $h \neq \varnothing$, $s^*(h, \theta_0)$ maximizes the buyer's expected payoff, and

(iii) the buyer's belief μ^* is updated according to Bayes' rule whenever possible.

Remark 1. Alba et al (1991) categorizes the consideration set procedure based on whether relevant choices are physically present at the time of choice. If choices are physically present, the consideration set may be determined by a stimulus such as a cue and a visual salience. On the other hand, if choices are not physically present, the consideration set may be determined by the decision maker's memory. In our model, the buyer's consideration set formation is not passively determined by a salience or memory. Instead, the buyer actively searches high quality products and her consideration set is determined by her search technology and the buyer's action. Since her search technology depends on factors such as her social network and online search engines, whether choices are physically present or not plays less important role in our setting.

[6] We focus on pure strategies since, whenever a mixed strategy equilibrium exists, the set of types who use the mixed strategy has measure zero.

Remark 2. There is another interpretation of our model. Unlike our basic setting, suppose that the buyer has already consumed x_1 but she forgets x_1 in the beginning of the game.[7] Then, the buyer searches high quality products in her "memory" and thus $\lambda(\theta)$ is defined as her recall probability of x_1 conditional on experience θ. Assumption 1 then means that when the buyer tries to recall high quality products she consumed, she recalls x_1 with higher probability if θ was higher. In this situation, the seller decides whether to remind her of the existence of x_1 without knowing whether she could recall x_1, i.e., reminder advertising.

3 Equilibrium

We start with the analysis of the buyer's behavior. Recall that if $(a, \omega) = (N, 0)$, then $X = \{x_0\}$ and the buyer chooses x_0. Thus, consider the case $X = \{x_0, x_1\}$. As we mentioned in the last section, there are three possible cases for $X = \{x_0, x_1\}$. The first case is $(a, \omega) = (M, 0)$. In this case, the buyer's consistent posterior belief about θ given strategy σ is

$$\mu_\sigma(\theta|M,0) = \frac{(1-\lambda(\theta))1_{\{\theta''|\sigma(\theta'')=M\}}(\theta)f(\theta)}{\int_{\theta'}(1-\lambda(\theta'))1_{\{\theta''|\sigma(\theta'')=M\}}(\theta')f(\theta')d\theta'}$$

where $1_{\{\theta''|\sigma(\theta'')=M\}}(\theta')$ is the indicator function of $\{\theta''|\sigma(\theta'') = M\}$.

The second case is $(a, \omega) = (M, 1)$. In this case, her consistent posterior belief given strategy σ is

$$\mu_\sigma(\theta|M,1) = \frac{\lambda(\theta)1_{\{\theta''|\sigma(\theta'')=M\}}(\theta)f(\theta)}{\int_{\theta'}\lambda(\theta')1_{\{\theta''|\sigma(\theta'')=M\}}(\theta')f(\theta')d\theta'}.$$

The third case is $(a, \omega) = (N, 1)$. Her consistent posterior belief given strategy σ is then

$$\mu_\sigma(\theta|N,1) = \frac{\lambda(\theta)1_{\{\theta''|\sigma(\theta'')=N\}}(\theta)f(\theta)}{\int_{\theta'}\lambda(\theta')1_{\{\theta''|\sigma(\theta'')=N\}}(\theta')f(\theta')d\theta'}$$

where $1_{\{\theta''|\sigma(\theta'')=N\}}(\theta')$ is the indicator function of $\{\theta''|\sigma(\theta'') = N\}$.

Then, the expected value of θ conditional on (a, ω) is

$$\theta_\sigma(a,\omega) = \int \theta\mu_\sigma(\theta|a,\omega)d\theta.$$

When $X = \{x_0, x_1\}$, the buyer compares $\theta_\sigma(a, \omega)$ with θ_0 and chooses one with a higher

[7] Shapiro (2006) analyzes advertising with consumers who forget their consumption experiences.

value. Thus, given σ, her optimal reaction is

$$s(h,\theta_0|\sigma)=\begin{cases} x_1 & \text{if } h\neq\varnothing \text{ and } \theta_0\leq\theta_\sigma(a,\omega) \\ x_0 & \text{if } h\neq\varnothing \text{ and } \theta_0>\theta_\sigma(a,\omega) \\ x_0 & \text{if } h=\varnothing \end{cases}.$$

Turning to the seller's problem, let $U_S^g(a,\theta)$ denote type θ seller's expected payoff from a given the buyer's optimal reaction and σ. Since the seller's payoff from selling x_1 is 1, his expected payoff from a is the probability that the buyer chooses x_1 conditional on θ and a, that is,

$$U_S^g(a,\theta)=\begin{cases} \lambda(\theta)G(\theta_\sigma(M,1))+(1-\lambda(\theta))G(\theta_\sigma(M,0)) & \text{if } a=M \\ \lambda(\theta)G(\theta_\sigma(N,1)) & \text{if } a=N \end{cases}.$$

In the rest of this section, we analyze the existence of equilibria and characterize them. A pooling equilibrium is **M-pooling** if $\sigma(\theta)=M$ for all θ in the equilibrium. Similarly, a pooling equilibrium is **N-pooling** if $\sigma(\theta)=N$ for all θ in the equilibrium.

Recall that the seller's payoff function is independent of the seller's action a. Thus, it is easy to see that uninformative equilibria, i.e., the M-pooling and N-pooling equilibria, always exist.[8] Then, the first question is the existence of an equilibrium where the equilibrium strategy depends on θ, i.e., informative equilibrium. To investigate this question, consider a class of partial pooling strategies. A strategy is a **cutoff countersignaling** if

$$\sigma(\theta)=\begin{cases} N \text{ if } \theta>\hat{\theta} \\ M \text{ if } \theta<\hat{\theta} \end{cases}.$$

In short, the seller sends a message if the value of his product is *lower* than a certain level. On the other hand, he remains silent if the value of his product is *higher* than the certain level. An equilibrium is a **cutoff countersignaling equilibrium** if he uses a cutoff countersignaling strategy in the equilibrium.

The next proposition states that some informative equilibrium can exist even though the seller's payoff function is independent of his action and type.[9]

[8] In Section 5, we refine the pooling equilibria.

[9] Recall that the standard cheap talk game has informative equilibria since the sender's payoff function depends on the type.

Proposition 1. *There exists a cutoff countersignaling equilibrium if $\lambda(\theta)$ is sufficiently low for some $\theta \in (\theta_{\min}, \theta_{\max}]$. On the other hand, there is no cutoff countersignaling equilibrium if $\lambda(\theta)$ is sufficiently high for any θ.*

Proof. See appendix.

An intuition of the first part of Proposition 1 is as follows. Suppose that the seller plays a cutoff countersignaling strategy. If $\lambda(\theta)$ is sufficiently low for some type and lower types remain silent pretending higher types, the buyer rarely chooses x_1 since it is rarely noticed, i.e., $\lambda(\theta)$ is low. On the other hand, higher types know that the buyer finds x_1 with higher probability even if he remains silent. Hence, higher types can differentiate themselves from lower types by remaining silent. Thus, in this equilibrium, the buyer interprets the silence as a signal of "confidence," while messaging as a signal of "lack of confidence." For an intuition of the second part, suppose that the seller uses a cutoff countersignaling strategy even though the buyer can find x_1 with a high probability for any θ. Then, since the buyer finds the product with a high probability even if the seller remains silent, a low type has an incentive to remain silent pretending a higher type.

In the informative equilibrium, the equilibrium cutoff type is determined so that the cutoff type's expected payoff from sending a message and that from remaining silent are indifferent given the cutoff countersignaling strategy. The existence of such a cutoff, i.e., a fixed point, is guaranteed if $\lambda(\theta)$ is sufficiently low for some $\theta > \theta_{\min}$. Then, by the increasingness of $\lambda(\theta)$, all types who are higher than the cutoff type prefer to remain silent, while all types who are lower than the cutoff type prefer to send a message. On the other hand, if $\lambda(\theta)$ is sufficiently high for any θ, such a fixed point does not exist and thus there is no cutoff countersignaling equilibrium.

The next question is whether other classes of informative equilibria can exist. The following proposition states that the answer is no.

Proposition 2. *Any informative equilibrium is a cutoff countersignaling equilibrium.*

We prove Proposition 2 by establishing the following lemma.

Lemma 1. *There is no equilibrium in which $\sigma(\theta') = N$ and $\sigma(\theta'') = M$ if $\theta' < \theta''$.*

Proof. See appendix.

Since it is difficult to provide an intuition for Lemma 1, we give a sketch of the proof. Suppose that there exists an equilibrium such that $\sigma(\theta') = N$ and $\sigma(\theta'') = M$ if $\theta' < \theta''$. Observe the probability that the buyer chooses x_1 given (a, ω) does not depend on θ given a strategy. Hence, given a strategy, θ affects the seller's payoff only through the search technology $\lambda(\theta)$. Hence, whenever $\omega = 1$, the ratio of the seller's expected payoff from M to that from N is constant in θ. On the other hand, the expected payoff from M is decreasing in θ if $\omega = 0$, while the expected payoff from N is always 0. Hence, given any strategy σ, the ratio of the expected payoff from M to that from N is decreasing in θ. Hence, whenever θ' prefers N to M, θ'' also prefers N to M.

Remark 3. Even though an intuition behind the countersignaling result in our model is similar to that in Feltovich et al (2002), the equilibrium structure of our game is entirely different from theirs. In their model, the single crossing property of the payoff function plays a crucial role, while our sender's payoff function is independent of his type and his choice of a signal. Thus, unlike our model, their game has a separating equilibrium in addition to the countersignaling equilibrium.

Remark 4. We can consider a richer message space for the seller, i.e., $A = \{N, M_1, M_2, ..\}$ where $\{M_1, M_2, ..\}$ can be interpreted as a set of "sales pitches." However, as long as each sales pitch can make the buyer aware of x_1, any equilibrium is "sales pitch neutral," that is, if there exists an equilibrium in which M' is chosen with a strictly positive probability, while M'' is never chosen, we can construct an equilibrium with the same equilibrium payoff by replacing M' with M''. To see the reason recall that the seller's payoff is independent of his type and the choice of a sales pitch. Hence, whenever M' is more profitable than M'' for one type, M' is also more profitable than M'' for any other types. Hence, any two sales pitches are equivalent in terms of informativeness when the seller uses them in equilibrium.

In the rest of this section, we analyze how the change of the search technology affects the equilibrium cutoff type. For the sake of this comparative statics, we focus on the case in which there exists a unique informative equilibrium. To parameterize the search technology, let $\widetilde{\lambda}(\theta) = \delta\lambda(\theta)$ where $\delta \in (0, 1/\lambda(\theta_{\min}))$. Thus, if $\delta > 1$, the search technology $\widetilde{\lambda}(\theta)$ finds x_1 with higher probability than $\lambda(\theta)$ for each θ. On the other hand, if $\delta < 1$, the search technology $\widetilde{\lambda}(\theta)$ finds x_1 with lower probability than $\lambda(\theta)$ for each θ.

Let $\hat{\theta}|_{\lambda'}$ be the unique equilibrium cutoff under search technology λ' and $U_S^{\sigma_{\hat{\theta}}}(M, \hat{\theta})_{\lambda'}$ denote the cutoff type's expected payoff from a given cutoff countersignaling strategy $\sigma_{\hat{\theta}}$ and search technology λ'.

Note that

$$\frac{U_S^{\sigma_{\hat{\theta}}}(M, \hat{\theta})_\lambda}{U_S^{\sigma_{\hat{\theta}}}(N, \hat{\theta})_\lambda} = \frac{\lambda(\hat{\theta})G(\theta_{\sigma_{\hat{\theta}}}(M, 1)) + (1 - \lambda(\hat{\theta}))G(\theta_{\sigma_{\hat{\theta}}}(M, 0))}{\lambda(\hat{\theta})G(\theta_{\sigma_{\hat{\theta}}}(N, 1))}.$$

First, observe that if $\delta \in (0, 1)$, then $\frac{U_S^{\sigma_{\hat{\theta}|_\lambda}}(M, \hat{\theta}|_\lambda)_\lambda}{U_S^{\sigma_{\hat{\theta}|_\lambda}}(N, \hat{\theta}|_\lambda)_\lambda} = 1 < \frac{U_S^{\sigma_{\hat{\theta}|_\lambda}}(M, \hat{\theta}|_\lambda)_{\delta\lambda}}{U_S^{\sigma_{\hat{\theta}|_\lambda}}(N, \hat{\theta}|_\lambda)_{\delta\lambda}}$. In this case, it is easy to show that whenever there exists a unique informative equilibrium, we have $\hat{\theta}|_{\delta\lambda} < \hat{\theta}|_\lambda$. That is, the set of types who send a message in the equilibrium becomes smaller.

Second, note that if $\delta \in (1, 1/\lambda(\theta_{\min}))$, then $\frac{U_S^{\sigma_{\hat{\theta}|_\lambda}}(M, \hat{\theta}|_\lambda)_\lambda}{U_S^{\sigma_{\hat{\theta}|_\lambda}}(N, \hat{\theta}|_\lambda)_\lambda} = 1 > \frac{U_S^{\sigma_{\hat{\theta}|_\lambda}}(M, \hat{\theta}|_\lambda)_{\delta\lambda}}{U_S^{\sigma_{\hat{\theta}|_\lambda}}(N, \hat{\theta}|_\lambda)_{\delta\lambda}}$. In this case, we can show that $\hat{\theta}|_{\delta\lambda} > \hat{\theta}|_\lambda$ whenever a unique informative equilibrium exists. Thus, the set of types who send a message in the equilibrium becomes larger.

4 Equilibrium payoff comparison

As other signaling games, our game also has multiple equilibria. This section analyzes which equilibrium is best for each player. Let $\theta_\sigma(a, \omega)$ denote the buyer's expected value of x_1 conditional on (a, ω) given a strategy σ.

First, note that

$$\theta_{\sigma_{\hat{\theta}}}(N, 1) = \int_{\theta > \hat{\theta}} \theta \frac{\lambda(\theta) f(\theta)}{\int_{\theta' > \hat{\theta}} \lambda(\theta') f(\theta') d\theta'} d\theta$$

where $\sigma_{\hat{\theta}}$ is the cutoff countersignaling strategy with $\hat{\theta}$.

Second, let σ_{NP} be the N-pooling strategy and σ_{MP} be the M-pooling strategy. Observe that

$$\theta_{\sigma_{NP}}(N, 1) = \theta_{\sigma_{MP}}(M, 1) = \int_\theta \theta \frac{\lambda(\theta) f(\theta)}{\int_{\theta'} \lambda(\theta') f(\theta') d\theta'} d\theta.$$

Then, it is easy to see that $\theta_{\sigma_{\hat{\theta}}}(N, 1) > \theta_{\sigma_{NP}}(N, 1) = \theta_{\sigma_{MP}}(M, 1)$.

Finally, note that

$$\theta_{\sigma_{MP}}(M,0) = \int_{\theta} \theta \frac{(1-\lambda(\theta))f(\theta)}{\int_{\theta'}(1-\lambda(\theta'))f(\theta')d\theta'} d\theta.$$

Since $\lambda(\theta)$ is increasing in θ, $\theta_{\sigma_{NP}}(N,1) > \theta_{\sigma_{MP}}(M,0)$.

The next proposition clarifies how the buyer's best equilibrium depends on θ_0 and $\lambda(.)$

Proposition 3. *Suppose there exists a unique cutoff countersignaling equilibrium.*

(i) If $\lambda(\theta_{\max})$ is sufficiently low, the M-pooling is the best equilibrium for the buyer with $\theta_0 < \theta_{\sigma_{MP}}(M,0)$.

(ii) The cutoff countersignaling equilibrium is the best equilibrium for the buyer with $\theta_0 \in (\theta_{\sigma_{NP}}(N,1), \theta_{\sigma_{\hat{\theta}}}(N,1)]$.

(iii) All equilibria are indifferent to the buyer with $\theta_0 > \theta_{\sigma_{\hat{\theta}}}(N,1)$.

Proof. See appendix.

The idea of Proposition 3 is as follows. If the buyer's type is $\theta_0 < \theta_{\sigma_{MP}}(M,0)$, she chooses product x_1 in any equilibrium whenever $x_1 \in X$. Hence, if the probability that she finds x_1 is sufficiently low for any θ, she prefers the equilibrium where the seller always sends a message, i.e., the M-pooling equilibrium. If $\theta_0 \in (\theta_{\sigma_{NP}}(N,1), \theta_{\sigma_{\hat{\theta}}}(N,1)]$, the buyer chooses product x_1 only in the cutoff countersignaling equilibrium. Since the choice set is larger and the decision is optimal given information, her expected payoff in the cutoff countersignaling equilibrium has to be higher than that in other pooling equilibria. Finally, if $\theta_0 > \theta_{\sigma_{\hat{\theta}}}(N,1)$, the buyer never chooses x_1 in any equilibrium. Thus, all equilibria are indifferent to her. When $\theta_0 \in (\theta_{\sigma_{MP}}(M,0), \theta_{\sigma_{NP}}(N,1))$, the buyer always prefers the M-pooling equilibrium to the N-pooling equilibrium. However, whether she prefers the cutoff countersignaling equilibrium to the M-pooling equilibrium is sensitive to the parameters.

The buyer's best equilibrium in the ex ante stage, i.e. before the buyer observes θ_0, also depends on the search technology. For instance, if the buyer often fails to find the seller's product for any θ, i.e., $\lambda(\theta_{\max})$ is low, the M-pooling is the buyer's ex ante best equilibrium. Note that this is contrary to Spencian signaling models in which the receiver always prefers an informative equilibrium to pooling equilibria. This is because the receiver pays nothing for the informativeness of the signal in Spencian signaling models, while the informativeness of the signal requires lower probability of $x_1 \in X$ and thus does cost the receiver in our signaling model.

On the other hand, if the search technology can sort out high quality products with a high probability, the informative equilibrium can be the buyer's ex ante best equilibrium. To see the idea, suppose the search technology follows a logistic function $\lambda(\theta) = \frac{1}{1+e^{-\beta(\theta-\widetilde{\theta})}}$ where $\widetilde{\theta} \in int(\Theta)$. Note that, for large β, $\lambda(\theta)$ is a good approximation of a step function with a jump at $\widetilde{\theta}$. Thus, the search technology sorts out a product with $\theta > \widetilde{\theta}$ more accurately if β is larger. By Proposition 1, the existence of a cutoff countersignaling equilibrium is guaranteed for a large β. Now, for simplicity, assume that distribution $G(\theta_0)$ and $F(\theta)$ are both uniform over Θ. Then, we can show that one of equilibrium cutoffs can be arbitrarily close to $\widetilde{\theta}$ for large β. In this cutoff countersignaling equilibrium, the buyer's consideration set includes x_1 with a high probability since the buyer finds x_1 with a high probability for most of $\theta > \widetilde{\theta}$. Thus, if β is large, in almost all cases, the buyer chooses a product from $\{x_0, x_1\}$ based on a signal which essentially tells her whether $\theta > \widetilde{\theta}$ or not. Hence, the buyer's ex ante expected payoff in the informative equilibrium is higher than that in the M-pooling if β is sufficiently large.

Turning to the seller's perspective, the next proposition clarifies that how the seller's best equilibrium depends on θ and $\lambda(.)$.

Proposition 4.

(i) The cutoff countersignaling equilibrium cannot be the best equilibrium for the seller with $\theta < \hat{\theta}$. *On the other hand, it is the best equilibrium for the seller with* $\theta_{\max}$ *if* $\lambda(\theta_{\max})$ *is sufficiently large.*

(ii) If $\lambda(\theta_{\max})$ *is sufficiently low, the* M*-pooling is the best equilibrium for any* θ.

Proof. See appendix.

To provide an intuition for (i), observe that when there exists an informative equilibrium, the lower types send a message in both the M-pooling equilibrium and the informative equilibrium. Hence, for the buyer, the expected value of x_1 conditional on the message is lower in the informative equilibrium. Thus, the lower types always prefer the M-pooling equilibrium. On the other hand, for $\theta > \hat{\theta}$, the expected value of x_1 conditional on $\omega = 1$ in the informative equilibrium is higher than that in any uninformative equilibrium. On the other hand, the expected value of x_1 conditional on $\omega = 0$ in the informative equilibrium is lower than that in the M-pooling equilibrium. Hence, if $\lambda(\theta_{\max})$ is sufficiently high and the probability of $\omega = 1$ is high, type $\theta_{\max}$ seller prefers the informative equilibrium to any uninformative equilibrium. By continuity, this argument holds for any seller with a sufficiently high θ.

To provide an intuition for (ii), suppose the probability that the buyer finds x_1 is very low even for $\theta_{\max}$. Then, if the seller remains silent, the buyer rarely notices the existence of x_1. Thus, the expected payoff in the M-pooling equilibrium in which the buyer always notices the existence of x_1 can be higher than in the informative equilibrium.

Remark 5. At first glance, the limitation of attention seems to be a "market friction" which makes the transaction less efficient. However, there is a positive aspect of this market friction. To see the point, suppose the buyer's attention is unlimited and she can identify all feasible products, i.e., $\lambda(\theta) = 1$ for any θ. Then, since the seller's message plays no role, all equilibria are uninformative and thus the buyer has to make a decision based on her prior probability, that is, the equilibrium payoff is the same as that in the M-pooling equilibrium. On the other hand, when the buyer's attention is limited and she can fail to be aware of some feasible products, whether to send a message becomes an effective signaling device for the seller. As we discussed earlier in this section, the informative equilibrium can be the buyer's best equilibrium in the ex ante stage. Thus, in such a case, the buyer and the seller with high θ can be better off when the buyer's attention is limited.

5 Refinement

This section refines the set of equilibria. Since the sender's payoff function is independent of the choice of a signal, "equilibrium dominance based refinements" are not effective. Thus, we employ **perfect sequential equilibrium (PSE)** introduced by Grossman and Perry (1991). PSE refines the set of perfect Bayesian equilibria by restricting off-the-equilibrium beliefs to be "credible." That is, once a deviation has occurred, the receiver tries to rationalize the deviation by trying to find a set of types $C \subset \Theta$ that would benefit from the deviation if it is thought C deviated, but $\theta \notin C$ loses from the deviation. More preciously, suppose the seller chooses an off-the-equilibrium action. The receiver then tries to find $C \subset \Theta$ such that, if the buyer chooses the optimal action believing the sender's type is in C, the set of types whose expected payoffs are strictly higher than the equilibrium payoff is exactly C. If such C exists, the **credible updating rule** given off-the-equilibrium action a' is

$$\mu_\sigma(\theta|a',\omega) = \begin{cases} \frac{\lambda(\theta)f(\theta)}{\int_{\theta' \in C} \lambda(\theta')f(\theta')d\theta'} \text{ if } \theta \in C \text{ and } \omega = 1 \\ \frac{(1-\lambda(\theta))f(\theta)}{\int_{\theta' \in C}(1-\lambda(\theta'))f(\theta')d\theta'} \text{ if } \theta \in C \text{ and } \omega = 0 \\ \mu_\sigma(\theta|a',\omega) = 0 \text{ if } \theta \notin C. \end{cases}$$

An equilibrium is a PSE if the sender has no incentive to deviate under the credible updating rule. In a PSE, the updating rule has to follow Bayes' rule whenever possible. Thus, whenever there is no off-the-equilibrium action in a PBE, the equilibrium is always a PSE. Hence, *any cutoff countersignaling equilibrium is a PSE* in our model. Our question is therefore whether each pooling equilibrium is a PSE.

Proposition 5.

(i) *The N-pooling equilibrium is not a PSE.*

(ii) *If $\lambda(\theta_{\max})$ is sufficiently large and a cutoff countersignaling equilibrium exists, the M-pooling equilibrium is not a PSE.*

Proof. See appendix.

Proposition 5 implies that all equilibria are informative if $\lambda(\theta_{\max})$ is sufficiently large and there exists some informative equilibrium. The idea of Proposition 5-(i) is as follows. Note that, since the seller can ensure that the buyer notices the product, the seller's expected payoff in the M-pooling equilibrium is always higher than that in the N-pooling equilibrium. Thus, whenever the buyer receives a message in the N-pooling equilibrium, he may believe that the seller can be any type. In fact, if the buyer makes his decision based on such a belief, all types have an incentive to deviate from the N-pooling equilibrium.

To provide an intuition for Proposition 5-(ii), suppose that, by choosing N, the seller with a high type tries to differentiate himself from lower types in the M-pooling equilibrium. When $\lambda(\theta_{\max})$ is high, the buyer may speculate that the seller who might try such a deviation should belong to a set of high types. In fact, when $\lambda(\theta_{\max})$ is sufficiently high and there exists a cutoff countersignaling equilibrium, we can always find a set of high types such that, when the buyer believes that the seller who deviated belongs to the set of high types, the seller has an incentive to deviate from the M-pooling equilibrium if and only if the seller's type belongs to the set of high types.

6 Extensions

This section provides two extensions. First, we investigate the robustness of our results with respect to a messaging cost. Second, we extend the model to a multi-seller setting.

6.1 Costly message

In our basic setting, there is no exogenous cost to send a message. However, since marketing is often costly, e.g., advertising, it is important to analyze the robustness of our result with respect to a messaging cost. To investigate the question, suppose that the seller has to pay $c > 0$ if $a = M$ and the seller's payoff from selling x_1 is $b \geq 1$. Then, let $\Gamma(b, c)$ be our signaling game given (b, c).

The following proposition states that the equilibrium structure can be preserved if c/b is sufficiently small.

Proposition 6. *Given b, suppose there exits a cutoff countersignaling equilibrium in $\Gamma(b, 0)$ and let $\sigma_{\hat{\theta}^*}$ be the equilibrium strategy. If c is such that*

$$\begin{aligned} \text{(i) } \frac{c}{b} &< \lambda(\theta_{\max})[G(\theta_{\sigma_{MP}}(M,1)) - 1] + (1 - \lambda(\theta_{\max}))G(\theta_{\sigma_{MP}}(M,0)), \\ \text{(ii) } \frac{c}{b} &< G(\theta_{\sigma_{\hat{\theta}^*}}(M,0)), \end{aligned}$$

then there exists a cutoff countersignaling equilibrium in $\Gamma(b, c)$. On the other hand, there is no cutoff countersignaling equilibrium if $\frac{c}{b} > G(\theta_{\sigma_{MP}}(M, 0))$.

Proof. *See appendix.*

For a large $\lambda(\theta_{\max})$, condition (i) cannot be satisfied for any $c > 0$. However, the existence of a cutoff countersignaling equilibrium is still guaranteed as long as c/b is sufficiently low, that is, condition (i) is not a necessary condition.

As another candidate of an equilibrium strategy in this costly messaging setting, we may consider a cutoff strategy in which only higher types send a message. Formally, define a *cutoff signaling strategy* as $\sigma(\theta) = M$ if $\theta > \hat{\theta}$ and $\sigma(\theta) = N$ if $\theta < \hat{\theta}$. We can show that if $c/b < G(\theta_{\sigma_{MP}}(M, 0))$, there is no cutoff signaling equilibrium. On the other hand, if $c/b > G(\theta_{\sigma_{MP}}(M, 0))$, a cutoff signaling equilibrium can exist for some parameters. Finally, given any c/b, if $\lambda(\theta_{\max})$ is sufficiently low, there is no cutoff signaling equilibrium.

6.2 Multi-seller

Suppose there are seller 1 and 2 who are ex ante identical, that is, the product quality of each seller is independently drawn from distribution $F(\theta)$. As the basic setting, let $\lambda(\theta_i)$ be the search technology given θ_i, i.e., the probability of finding x_i conditional on θ_i. Note that the search technology is symmetric in the sense that it depends only on the product quality. One interpretation of this setting is that two sellers are similar in terms of the brand power.

Now we specify the formation process of the consideration set. In the multi-seller setting, we define x_0 as the best competing product of x_1 and x_2. Then, θ_0 is drawn from $G(\theta_0)$ as the basic setting. How each seller can affect the buyer's consideration set is analogous to the basic setting. First, if both sellers send a message, i.e., $(a_1, a_2) = (M, M)$, the buyer becomes aware of both products.[10] Then, $X = \{x_0, x_1, x_2\}$ with probability 1. Second, if $(a_1, a_2) = (N, N)$, $x_i \in X$ when her search outcome is $\omega_i = 1$, while $x_i \notin X$ when her search outcome is $\omega_i = 0$. Then, since the probability of $\omega_i = 1$ condition on θ_i is $\lambda(\theta_i)$,

$$X = \begin{cases} \{x_0, x_1, x_2\} \text{ with } \lambda(\theta_1)\lambda(\theta_2) \\ \{x_0, x_1\} \text{ with } \lambda(\theta_1)(1 - \lambda(\theta_2)) \\ \{x_0, x_2\} \text{ with } (1 - \lambda(\theta_1))\lambda(\theta_2) \\ \{x_0\} \text{ with } (1 - \lambda(\theta_1))(1 - \lambda(\theta_2)) \end{cases}.$$

Third, if $(a_1, a_2) = (M, N)$, the buyer becomes aware of x_1, while whether she notices x_2 or not depends on her search outcome. Hence,

$$X = \begin{cases} \{x_0, x_1, x_2\} \text{ with } \lambda(\theta_2) \\ \{x_0, x_1\} \text{ with } 1 - \lambda(\theta_2) \end{cases}$$

[10] This setting is contrary to Eliaz and Spiegler (2011b) in which two products compete to get the buyer's attention. Note that their interest is in the process of "attention-grabbing," while our interest is in the process of "awareness." It is possible for the buyer to be aware of multiple products.

Similarly, if $(a_1, a_2) = (N, M)$,

$$X = \begin{cases} \{x_0, x_1, x_2\} \text{ with } \lambda(\theta_1) \\ \{x_0, x_2\} \text{ with } 1 - \lambda(\theta_1) \end{cases}$$

We focus on symmetric equilibria of the game, i.e., in equilibrium, $\sigma_i(\theta_i) = \sigma_j(\theta_j)$ whenever $\theta_i = \theta_j$. Note that when $(a_1, \omega_1) = (a_2, \omega_2) \neq (N, 0)$, the expected value of two products are the same for the buyer. Thus, we assume that if $(a_1, \omega_1) = (a_2, \omega_2)$ and the expected value of x_1 is higher than θ_0, the buyer chooses x_1 and x_2 with the same probability.

The following proposition states that our main results are preserved even if there are two sellers.

Proposition 7. *In the two-seller setting, there exists a cutoff countersignaling equilibrium if $\lambda(\theta)$ is sufficiently low for some $\theta \in (\theta_{\min}, \theta_{\max}]$. Moreover, any informative equilibrium is a cutoff countersignaling equilibrium.*

Proof. See appendix.

We can also show the same result with any finite number of sellers. On the other hand, it is not obvious how the number of sellers affects the equilibrium cutoff type. Note that, given a strategy, a larger number of sellers makes the seller's expected payoff from each action lower. Thus, the net effect of larger number of sellers on the equilibrium cutoff type depends on the parameters.

In general, it is not easy to analyze the case with ex ante heterogeneous sellers. On the other hand, an extreme case is relatively easy to analyze in our framework. Suppose seller 2 has a very strong brand and the buyer finds seller 2's product with probability 1 given any θ_2. On the other hand, seller 1 has a weaker brand and the buyer's search technology for x_1 satisfies Assumption 1. In this case, it is easy to see that there is no equilibrium in which seller 2's strategy depends on θ, that is, seller 2 uses a pooling strategy in any equilibrium. Then, seller 1's equilibrium strategy is analogous to that in the basic setting since seller 1 can treat x_2 as x_0 in the basic setting.

7 Discussion

So far, we analyze the situation in which silence can be more persuasive than an attention-grabbing marketing. Based on our framework, this section discusses situations in which an attention-grabbing marketing can be an effective strategy for any type.

7.1 Mature vs. New product

Our model implicitly assumes that the seller's product is relatively mature. To see the point, suppose the seller is introducing a new product in the market. Then, since there is no consumer who knows the product, neither her friend's suggestion nor a search engine reflects the quality of the product. Thus, for a new product, the buyer's search technology may be independent of the product quality, that is, a violation of Assumption 1.

It is easy to show that if the search technology is independent of the product quality, any generic equilibrium in our model is uninformative. In other words, the M-pooling is a reasonable equilibrium prediction when the seller's product is new in the market. This prediction is consistent with the casual observation of new product campaigns. A company launches a campaign for a new product even if the product quality is high.

7.2 Direct vs. Mass marketing

Let q be the probability that the seller's message can make the buyer aware of the product. Note that, in our basic model, q is assumed to be 1. Such an assumption may be reasonable for a direct marketing strategy. For example, when a salesperson approaches a customer to introduce his product, i.e., direct selling, it is easy to make her aware of his product. On the other hand, if a firm advertises a product through a mass media, i.e., mass marketing, the probability that the buyer listens or watches his advertisement can be low, i.e., a small q.

It is easy to show that our main result is preserved as long as q is sufficiently high. On the other hand, there is no informative equilibrium if q is sufficiently small given the buyer's search technology. Thus, when we analyze a mass marketing strategy with a low q, a reasonable equilibrium prediction may be the M-pooling equilibrium, that is, the seller advertises his product even if its quality is high.

7.3 Experience goods vs. Search goods

Nelson (1974) categorizes goods into two types to analyze informative advertising. The first type is *experience goods* whose quality is hard to observe in advance, e.g., foods, drinks, and books. The second type is *search goods* which can be easily evaluated by observable features and characteristics, e.g., clothes and furniture. Since the product quality is the seller's private information in our model, our model implicitly assumes that product x_1 is an experience good. On the other hand, if the seller's product is a search good and the buyer can observe θ, the seller does not need to take into account the signaling effect of his marketing strategy. Hence, for search goods, the goal of the marketing strategy can be simply "standing out."

8 Summary

This paper analyzed how an attention-grabbing marketing can "backfire" and when it can be effective. To investigate the question, we analyzed a signaling game in which the receiver can fail to notice some feasible choices and follows a consideration set procedure.

Even though the seller's payoff function is independent of his choice of a signal and type, we show that:

- If the search technology can filter out low quality products with a high probability, there exists a countersignaling equilibrium, that is, lower types use an attention-grabbing marketing, while higher types remain silent. Moreover, any informative equilibrium is a countersignaling equilibrium in our game.

Thus, silence is more "persuasive" than an attention-grabbing marketing in the informative equilibrium. Since the game also has uninformative equilibria, we compared the equilibrium payoffs. We then found the following.

- If the buyer's search technology can sort out high quality products with a high probability, the buyer's best equilibrium in the ex ante stage can be a countersignaling equilibrium.

That is, the possibility of failing to notice some feasible products makes the marketing strategy an effective signaling device. Thus, the buyer's "limitation of attention" can improve efficiency of the transaction in the adverse selection environment.

On the other hand,

- when the buyer's search technology often fails to find the seller's product even if its quality is very high, the buyer prefers a pooling equilibrium to any informative equilibrium.

Note that this is contrary to Spencian signaling models where the receiver prefers informative equilibria to pooling equilibria.

Finally, we obtain the following implications based on our framework.

- Silence can be more persuasive than an attention-grabbing marketing if (i) the seller's product is a relatively mature "experience good" and (ii) the marketing strategy can effectively make the buyer aware of his product, e.g., a direct selling.

- If the seller is promoting a new product or a "search good" through a mass media, an attention-grabbing marketing may be the effective strategy.

9 Appendix

This section provides the omitted proofs.

9.1 Proof of Proposition 1

Let $\sigma_{\hat{\theta}}$ be the cutoff countersignaling strategy with cutoff $\hat{\theta}$. Then, the expected value of x_1 given $(a, \omega) = (N, 1)$ and $\sigma_{\hat{\theta}}$ is

$$\theta_{\sigma_{\hat{\theta}}}(N, 1) = \int_{\theta > \hat{\theta}} \theta \frac{\lambda(\theta) f(\theta)}{\int_{\theta > \hat{\theta}} \lambda(\theta') f(\theta')} d\theta.$$

Note that the buyer never chooses x_1 if $(a, \omega) = (N, 0)$. Thus, type θ seller's expected payoff from N given $\sigma_{\hat{\theta}}$ is

$$U_S^{\hat{\theta}}(N, \theta) = \lambda(\theta) G(\theta_{\sigma_{\hat{\theta}}}(N, 1)).$$

On the other hand, let

$$\theta_{\sigma_{NP}}(N,1) = \int_{\theta} \theta \frac{\lambda(\theta)f(\theta)}{\int_{\theta'} \lambda(\theta')f(\theta')d\theta'} d\theta,$$

that is, this is the expected value of x_1 given $(a,\omega) = (N,1)$ when the seller uses the N-pooling strategy.

Note that

$$\lim_{\hat{\theta} \to \theta_{\min}} U_S^{\hat{\theta}}(N,\hat{\theta}) = \lambda(\theta_{\min})G(\theta_{\sigma_{NP}}(N,1)) > 0.$$

On the other hand, the expected value of x_1 given $(a,\omega) = (M,\omega)$ and $\sigma_{\hat{\theta}}$ is

$$\theta_{\sigma_{\hat{\theta}}}(M,\omega) = \begin{cases} \int_{\theta<\hat{\theta}} \theta \frac{\lambda(\theta)f(\theta)}{\int_{\theta'<\hat{\theta}} \lambda(\theta')f(\theta')d\theta'} d\theta \text{ if } \omega = 1 \\ \int_{\theta<\hat{\theta}} \theta \frac{(1-\lambda(\theta))f(\theta)}{\int_{\theta<\hat{\theta}}(1-\lambda(\theta'))f(\theta')d\theta'} d\theta \text{ if } \omega = 0 \end{cases}.$$

Hence, type θ seller's expected payoff from M given $\sigma_{\hat{\theta}}$ is

$$U_S^{\sigma_{\hat{\theta}}}(M,\theta) = \lambda(\theta)G(\theta_{\sigma_{\hat{\theta}}}(M,1)) + (1-\lambda(\theta))G(\theta_{\sigma_{\hat{\theta}}}(M,0)).$$

Moreover, observe that $\lim_{\hat{\theta}\to\theta_{\min}} G(\theta_{\sigma_{\hat{\theta}}}(M,1)) = \lim_{\hat{\theta}\to\theta_{\min}} G(\theta_{\sigma_{\hat{\theta}}}(M,0)) = 0$. Hence,

$$\lim_{\hat{\theta}\to\theta_{\min}} U_S^{\sigma_{\hat{\theta}}}(M,\hat{\theta}) = 0 < \lim_{\hat{\theta}\to\theta_{\min}} U_S^{\hat{\theta}}(N,\hat{\theta}).$$

Now, fix any $\hat{\theta}' \in (\theta_{\min}, \theta_{\max}]$. Note that, by choosing a low search technology for $\hat{\theta}'$, we can make $U_S^{\hat{\theta}'}(N,\hat{\theta}')$ arbitrarily close to 0. On the other hand, since $\lambda(\theta)$ is strictly increasing in θ, we have $G(\theta_{\sigma_{\hat{\theta}}}(M,1)) > G(\theta_{\sigma_{\hat{\theta}}}(M,0))$. Then, $U_S^{\sigma_{\hat{\theta}}}(M,\hat{\theta}') \geq G(\theta_{\sigma_{\hat{\theta}}}(M,0)) > 0$ for any search technology and thus $U_S^{\sigma_{\hat{\theta}'}}(M,\hat{\theta}') > U_S^{\sigma_{\hat{\theta}'}}(N,\hat{\theta}')$ for a sufficiently low search technology $\lambda(\hat{\theta}')$. Observe that $\theta_{\sigma_{\hat{\theta}}}(N,1)$, $\theta_{\sigma_{\hat{\theta}}}(M,1)$ and $\theta_{\sigma_{\hat{\theta}}}(M,0)$ are continuous in $\hat{\theta}$. Thus, both $U_S^{\hat{\theta}}(N,\hat{\theta})$ and $U_S^{\sigma_{\hat{\theta}}}(M,\hat{\theta})$ are also continuous in $\hat{\theta}$. Then, since $\lim_{\hat{\theta}\to\theta_{\min}} U_S^{\sigma_{\hat{\theta}}}(M,\hat{\theta}) < \lim_{\hat{\theta}\to\theta_{\min}} U_S^{\hat{\theta}}(N,\hat{\theta})$, the continuity guarantees the existence of $\hat{\theta}^* \in (\theta_{\min}, \hat{\theta}')$ such that $U_S^{\sigma_{\hat{\theta}}}(M,\hat{\theta}^*) = U_S^{\sigma_{\hat{\theta}}}(N,\hat{\theta}^*)$.

We claim that the cutoff countersignaling with cutoff $\hat{\theta}^*$ is an equilibrium strategy. To see the claim, note that

$$\begin{aligned} \frac{U_S^{\sigma_{\hat{\theta}^*}}(M,\theta)}{U_S^{\sigma_{\hat{\theta}^*}}(N,\theta)} &= \frac{\lambda(\theta)G(\theta_{\sigma_{\hat{\theta}^*}}(M,1)) + (1-\lambda(\theta))G(\theta_{\sigma_{\hat{\theta}^*}}(M,0))}{\lambda(\theta)G(\theta_{\sigma_{\hat{\theta}^*}}(N,1))} \\ &= \frac{G(\theta_{\sigma_{\hat{\theta}^*}}(M,1)) + \frac{1-\lambda(\theta)}{\lambda(\theta)}G(\theta_{\sigma_{\hat{\theta}^*}}(M,0))}{G(\theta_{\sigma_{\hat{\theta}^*}}(N,1))} \end{aligned}$$

Since $\frac{1-\lambda(\theta)}{\lambda(\theta)}$ is strictly decreasing in θ, $\frac{U_S^{\sigma_{\hat{\theta}^*}}(M,\theta)}{U_S^{\sigma_{\hat{\theta}^*}}(N,\theta)}$ is also strictly decreasing in θ. Then, since $\frac{U_S^{\sigma_{\hat{\theta}^*}}(M,\hat{\theta}^*)}{U_S^{\sigma_{\hat{\theta}^*}}(N,\hat{\theta}^*)} = 1$, $\frac{U_S^{\sigma_{\hat{\theta}^*}}(M,\theta')}{U_S^{\sigma_{\hat{\theta}^*}}(N,\theta')} < (>)1$ for $\theta' > (<)\hat{\theta}^*$, that is, no type has incentive to deviate given $\sigma_{\hat{\theta}^*}$. This establishes the first part of Proposition 1.

To show the second part of Proposition 1, note that, since $\lambda(\theta)$ is strictly increasing in θ, $G(\theta_{\sigma_{\hat{\theta}}}(N,1)) > G(\theta_{\sigma_{\hat{\theta}}}(M,1)) > G(\theta_{\sigma_{\hat{\theta}}}(M,0))$. Thus, when $\lambda(\theta_{\min})$ is sufficiently high, $U_S^{\sigma_{\hat{\theta}}}(M,\hat{\theta}) < U_S^{\sigma_{\hat{\theta}}}(N,\hat{\theta})$ for any $\hat{\theta}$, that is, there is no cutoff type which can make two actions indifferent for the seller. Q.E.D.

9.2 Proof of Lemma 1

Suppose there exists θ', θ'' such that $\sigma(\theta') = N$, $\sigma(\theta'') = M$ and $\theta' < \theta''$ in an equilibrium. Then, the following conditions have to be satisfied. Given the equilibrium strategy σ,

$$\begin{aligned}
\lambda(\theta')G(\theta_\sigma(N,1)) &\geq \lambda(\theta')G(\theta_\sigma(M,1)) + (1-\lambda(\theta'))G(\theta_\sigma(M,0)),\\
\lambda(\theta'')G(\theta_\sigma(N,1)) &\leq \lambda(\theta'')G(\theta_\sigma(M,1)) + (1-\lambda(\theta''))G(\theta_\sigma(M,0)).
\end{aligned}$$

By rearranging the condition, we have

$$\frac{\lambda(\theta'')G(\theta_\sigma(M,1)) + (1-\lambda(\theta''))G(\theta_\sigma(M,0))}{\lambda(\theta'')G(\theta_\sigma(N,1))} \geq 1 \geq \frac{\lambda(\theta')G(\theta_\sigma(M,1)) + (1-\lambda(\theta'))G(\theta_\sigma(M,0))}{\lambda(\theta')G(\theta_\sigma(N,1))}.$$

that is,

$$\frac{G(\theta_\sigma(M,1)) + \frac{1-\lambda(\theta'')}{\lambda(\theta'')}G(\theta_\sigma(M,0))}{G(\theta_\sigma(N,1))} \geq 1 \geq \frac{G(\theta_\sigma(M,1)) + \frac{1-\lambda(\theta')}{\lambda(\theta')}G(\theta_\sigma(M,0))}{G(\theta_\sigma(N,1))}.$$

However, since $\lambda(\theta)$ is strictly increasing in θ, $\frac{1-\lambda(\theta'')}{\lambda(\theta'')} < \frac{1-\lambda(\theta')}{\lambda(\theta')}$, a contradiction. Q.E.D.

9.3 Proof of Proposition 3

To prove (i), suppose the buyer's type is $\theta_0 < \theta_{\sigma_{MP}}(M,0)$. The buyer chooses x_1 with probability 1 in the M-pooling equilibrium. Thus, the buyer's expected payoff in the equilibrium is

$$U_B^{MP}(\theta_0) = \int [\lambda(\theta)\theta_{\sigma_{MP}}(M,1) + (1-\lambda(\theta))\theta_{\sigma_{MP}}(M,0)]f(\theta)d\theta.$$

On the other hand, in the N-pooling equilibrium, the buyer chooses x_1 with probability $\lambda(\theta)$, while she chooses x_0 with probability $1-\lambda(\theta)$. Thus, her expected payoff in the N-pooling equilibrium is

$$U_B^{NP}(\theta_0) = \int [\lambda(\theta)\theta_{\sigma_{NP}}(N,1) + (1-\lambda(\theta))\theta_0] f(\theta) d\theta.$$

Since $\theta_0 < \theta_{\sigma_{MP}}(M,0)$, $U_B^{MP}(\theta_0) > U_B^{NP}(\theta_0)$.

Next, in the cutoff countersignaling equilibrium with $\hat{\theta}$, the buyer chooses x_1 with probability $\lambda(\theta)$, while she chooses x_0 with probability $1-\lambda(\theta)$ if $\theta \geq \hat{\theta}$. Thus, her expected payoff in the cutoff countersignaling equilibrium is

$$\begin{aligned} U_B^{\hat{\theta}}(\theta_0) &= \int_{\theta \geq \hat{\theta}} [\lambda(\theta)\theta_{\sigma_{\hat{\theta}}}(N,1) + (1-\lambda(\theta))\theta_0] f(\theta) d\theta \\ &+ \int_{\theta < \hat{\theta}} \left[\begin{array}{c} \lambda(\theta) \max\left\{\theta_{\sigma_{\hat{\theta}}}(M,1), \theta_0\right\} \\ +(1-\lambda(\theta)) \max\left\{\theta_{\sigma_{\hat{\theta}}}(M,0), \theta_0\right\} \end{array} \right] f(\theta) d\theta. \end{aligned}$$

Note that $\theta_{\sigma_{\hat{\theta}}}(M,0) < \theta_{\sigma_{MP}}(M,0)$. Then, since $\theta_0 < \theta_{\sigma_{MP}}(M,0)$, $U_B^{MP}(\theta_0) > U_B^{\hat{\theta}}(\theta_0)$ for sufficiently low $\lambda(\theta_{\max})$. This establishes (i).

To show (ii), suppose the buyer's type is $\theta_0 \in (\theta_{\sigma_{NP}}(N,1), \theta_{\sigma_{\hat{\theta}}}(N,1)]$. Then, obviously, $U_B^{MP}(\theta_0) = U_B^{NP}(\theta_0) = \theta_0$. On the other hand, the buyer's expected payoff in the cutoff countersignaling equilibrium is

$$\begin{aligned} U_B^{\hat{\theta}}(\theta_0) &= \int_{\theta \geq \hat{\theta}} [\lambda(\theta)\theta_{\sigma_{\hat{\theta}}}(N,1) + (1-\lambda(\theta))\theta_0] f(\theta) d\theta \\ &+ \int_{\theta < \hat{\theta}} \theta_0 f(\theta) d\theta. \end{aligned}$$

Then, since $\theta_{\sigma_{\hat{\theta}}}(N,1) > \theta_0$, we have $U_B^{\hat{\theta}}(\theta_0) > U_B^{MP}(\theta_0) = U_B^{NP}(\theta_0)$ for $\theta_0 \in (\theta_{\sigma_{NP}}(N,1), \theta_{\sigma_{\hat{\theta}}}(N,1)]$.

To prove (iii), suppose the buyer's type is $\theta_0 > \theta_{\sigma_{\hat{\theta}}}(N,1)$. Then, obviously, $U_B^{\hat{\theta}}(\theta_0) = U_B^{MP}(\theta_0) = U_B^{NP}(\theta_0) = \theta_0$. Q.E.D.

9.4 Proof of Proposition 4

For (i), note that

$$\begin{aligned}
\theta_{\sigma_{\hat{\theta}}}(M,1) &= \int_{\theta<\hat{\theta}} \theta \frac{\lambda(\theta)f(\theta)}{\int_{\theta'<\hat{\theta}} \lambda(\theta')f(\theta')d\theta'} d\theta, \\
\theta_{\sigma_{\hat{\theta}}}(M,0) &= \int_{\theta<\hat{\theta}} \theta \frac{(1-\lambda(\theta))f(\theta)}{\int_{\theta<\hat{\theta}} (1-\lambda(\theta'))f(\theta')d\theta'} d\theta, \\
\theta_{\sigma_{MP}}(M,1) &= \int_{\theta} \theta \frac{\lambda(\theta)f(\theta)}{\int_{\theta'} \lambda(\theta')f(\theta')d\theta'} d\theta, \\
\theta_{\sigma_{MP}}(M,0) &= \int_{\theta} \theta \frac{(1-\lambda(\theta))f(\theta)}{\int_{\theta'} (1-\lambda(\theta'))f(\theta')d\theta'} d\theta.
\end{aligned}$$

Observe that $\theta_{\sigma_{\hat{\theta}}}(M,1) < \theta_{\sigma_{MP}}(M,1)$ and $\theta_{\sigma_{\hat{\theta}}}(M,0) < \theta_{\sigma_{MP}}(M,0)$. Thus, for $\theta < \hat{\theta}$,

$$\begin{aligned}
U_S^{\hat{\theta}}(\theta) &= \lambda(\theta)G(\theta_{\sigma_{\hat{\theta}}}(M,1)) + (1-\lambda(\theta))G(\theta_{\sigma_{\hat{\theta}}}(M,0)) \\
&< \lambda(\theta)G(\theta_{\sigma_{MP}}(M,1)) + (1-\lambda(\theta))G(\theta_{\sigma_{MP}}(M,0)) = U_S^{MP}(\theta)
\end{aligned}$$

That is, type $\theta < \hat{\theta}$ seller always prefers the M-pooling to any cutoff countersignaling equilibrium.

To show the second part of (i), note that, for any $\theta > \hat{\theta}$,

$$\begin{aligned}
\theta_{\sigma_{\hat{\theta}}}(N,1) &= \int_{\theta>\hat{\theta}} \theta \frac{\lambda(\theta)f(\theta)}{\int_{\theta'>\hat{\theta}} \lambda(\theta')f(\theta')d\theta'} d\theta, \\
\theta_{\sigma_{NP}}(N,1) &= \int_{\theta} \theta \frac{\lambda(\theta)f(\theta)}{\int_{\theta'} \lambda(\theta')f(\theta')d\theta'} d\theta
\end{aligned}$$

and

$$\begin{aligned}
U_S^{\hat{\theta}}(\theta) &= \lambda(\theta)G(\theta_{\sigma_{\hat{\theta}}}(N,1)) \\
U_S^{NP}(\theta) &= \lambda(\theta)G(\theta_{\sigma_{NP}}(N,1)) \\
U_S^{MP}(\theta) &= \lambda(\theta)G(\theta_{\sigma_{MP}}(M,1)) + (1-\lambda(\theta))G(\theta_{\sigma_{MP}}(M,0)),
\end{aligned}$$

Observe that

$$\theta_{\sigma_{\hat{\theta}}}(N,1) > \theta_{\sigma_{NP}}(N,1) = \theta_{\sigma_{MP}}(M,1) > \theta_{\sigma_{MP}}(M,0).$$

Hence, $U_S^{\hat{\theta}}(\theta) > U_S^{NP}(\theta)$ for $\theta > \hat{\theta}$. Moreover, if $\lambda(\theta_{\max})$ is sufficiently large, then $U_S^{\hat{\theta}}(\theta_{\max}) > U_S^{MP}(\theta_{\max})$.

To show Proposition 4-(ii), note that

$$\begin{aligned} U_S^{MP}(\theta) - U_S^{\hat{\theta}}(\theta) &= \lambda(\theta)[G(\theta_{\sigma_{MP}}(M,1) - G(\theta_{\sigma_{\hat{\theta}}}(N,1))] \\ &\quad +(1-\lambda(\theta))G(\theta_{\sigma_{MP}}(M,0). \end{aligned}$$

Since $G(\theta_{\sigma_{MP}}(M,0) > 0$, $U_S^{MP}(\theta) > U_S^{\hat{\theta}}(\theta)$ for sufficiently low $\lambda(\theta_{\max})$. On the other hand, $U_S^{MP}(\theta) > U_S^{NP}(\theta)$ since $U_S^{MP}(\theta) - U_S^{NP}(\theta) = (1-\lambda(\theta))G(\theta_{\sigma_{MP}}(M,0)) > 0$. Then, since $U_S^{MP}(\theta) > U_S^{\hat{\theta}}(\theta)$ for any $\theta < \hat{\theta}$ from (i), the M-pooling is the best equilibrium for the seller if $\lambda(\theta_{\max})$ is sufficiently small. Q.E.D.

9.5 Proof of Proposition 5

To prove (i), consider the N-pooling equilibrium. Suppose the seller deviates from the pooling equilibrium and chooses M. Let

$$\theta_C(M,\omega) = \int_\theta \theta\mu_C(\theta|M,\omega)d\theta$$

be the expected value of θ when the buyer believes that the seller who chooses M is in C. Then, the seller's expected payoff from this deviation given C is

$$U_S^C(\theta) = \lambda(\theta)G(\theta_C(M,1)) + (1-\lambda(\theta))G(\theta_C(M,0)).$$

Now, suppose $C = \Theta$. Then, the credible posterior belief is

$$\mu_C(\theta|M,\omega) = \begin{cases} \frac{\lambda(\theta)f(\theta)}{\int_{\theta'\in\Theta}\lambda(\theta')f(\theta')d\theta'} \\ \frac{(1-\lambda(\theta))f(\theta)}{\int_{\theta'\in\Theta}(1-\lambda(\theta'))f(\theta')d\theta'} \end{cases}$$

Then, since $\lambda(\theta)$ is strictly increasing in θ, $\theta_\Theta(M,0) < \theta_{\sigma_{NP}}(N,1) = \theta_\Theta(M,1)$. Thus,

$$\begin{aligned} U_S^\Theta(\theta) &= \lambda(\theta)G(\theta_\Theta(M,1)) + (1-\lambda(\theta))G(\theta_\Theta(M,0)) \\ &> \lambda(\theta)G(\theta_{\sigma_{NP}}(N,1)) = U_S^{NP}(\theta) \end{aligned}$$

for all θ. That is, any $\theta \in \Theta$ has incentive to deviate. Thus, the N-pooling is not PSE.

To prove (ii), consider the M-pooling equilibrium. Suppose that $\lambda(\theta_{\max})$ is sufficiently large so that

$$\lambda(\theta_{\max}) > \frac{G(\theta_{\sigma_{MP}}(M,0))}{1 - G(\theta_{\sigma_{MP}}(M,1)) + G(\theta_{\sigma_{MP}}(M,0))}.$$

Then,

$$\lim_{\hat{\theta}\to\theta_{\max}} \lambda(\hat{\theta})G(\theta_{\sigma_{\hat{\theta}}}(N,1)) > \lambda(\theta_{\max})G(\theta_{\sigma_{MP}}(M,1)) + (1-\lambda(\theta_{\max}))G(\theta_{\sigma_{MP}}(M,0)).$$

Suppose there exists a cutoff countersignaling equilibrium with cutoff $\hat{\theta}^*$. From the equilibrium condition,

$$\lambda(\hat{\theta}^*)G(\theta_{\sigma_{\hat{\theta}^*}}(N,1)) = \lambda(\hat{\theta}^*)G(\theta_{\sigma_{\hat{\theta}^*}}(M,1)) + (1-\lambda(\hat{\theta}^*))G(\theta_{\sigma_{\hat{\theta}^*}}(M,0)).$$

Then, since

$$\begin{aligned} & \lambda(\hat{\theta}^*)G(\theta_{\sigma_{\hat{\theta}^*}}(M,1)) + (1-\lambda(\hat{\theta}^*))G(\theta_{\sigma_{\hat{\theta}^*}}(M,0)) \\ < \ & \lambda(\hat{\theta}^*)G(\theta_{\sigma_{MP}}(M,1)) + (1-\lambda(\hat{\theta}^*))G(\theta_{\sigma_{MP}}(M,0)), \end{aligned}$$

the continuity guarantees the existence of $\hat{\theta}' \in (\hat{\theta}^*, \theta_{\max})$ such that

$$\lambda(\hat{\theta}')G(\theta_{\sigma_{\hat{\theta}'}}(N,1)) = \lambda(\hat{\theta}')G(\theta_{\sigma_{MP}}(M,1)) + (1-\lambda(\hat{\theta}'))G(\theta_{\sigma_{MP}}(M,0)).$$

Now, suppose if the seller deviates from the pooling equilibrium and chooses N, the buyer believes that his type is in $C = (\hat{\theta}', \theta_{\max}]$. Then, the credible posterior belief has to be

$$\mu_C(\theta|N,1) = \begin{cases} \frac{\lambda(\theta)f(\theta)}{\int_{\theta>\hat{\theta}'}\lambda(\theta')f(\theta')} \text{ if } \theta \geq \hat{\theta}' \\ 0 \text{ if } \theta < \hat{\theta}' \end{cases}.$$

By construction of $\hat{\theta}'$,

$$\begin{aligned} \frac{U_S^{MP}(\hat{\theta}')}{U_S^{(\hat{\theta}',\theta_{\max}]}(N,\hat{\theta}')} &= \frac{\lambda(\hat{\theta}')G(\theta_{\sigma_{MP}}(M,1)) + (1-\lambda(\hat{\theta}'))G(\theta_{\sigma_{MP}}(M,0))}{\lambda(\hat{\theta}')G(\theta_{(\hat{\theta}',\theta_{\max}]}(N,1))} \\ &= \frac{G(\theta_{\sigma_{MP}}(M,1)) + \frac{1-\lambda(\hat{\theta}')}{\lambda(\hat{\theta}')}G(\theta_{\sigma_{MP}}(M,0))}{G(\theta_{(\hat{\theta}',\theta_{\max}]}(N,1))} = 1 \end{aligned}$$

Then, since $\frac{1-\lambda(\theta)}{\lambda(\theta)}$ is strictly decreasing in θ,

$$\begin{aligned} \frac{U_S^{MP}(\theta)}{U_S^{(\hat{\theta}',\theta_{\max}]}(N,\theta)} &\leq 1 \text{ if } \theta \in C \\ \frac{U_S^{MP}(\theta)}{U_S^{(\hat{\theta}',\theta_{\max}]}(N,\theta)} &> 1 \text{ if } \theta \notin C \end{aligned}$$

That is, any $\theta \in C$ has incentive to deviate, while any $\theta \notin C$ has no incentive to deviate. Q.E.D.

9.6 Proof of Proposition 6

If sending a message costs $c > 0$, the seller's expected payoff from M given $\sigma_{\hat{\theta}}$ is

$$U_S^{\sigma_{\hat{\theta}}}(M, \theta) = b[\lambda(\theta)G(\theta_{\sigma_{\hat{\theta}}}(M, 1)) + (1 - \lambda(\theta))G(\theta_{\sigma_{\hat{\theta}}}(M, 0))] - c.$$

Moreover, observe that $\lim_{\hat{\theta}\to\theta_{\min}} G(\theta_{\sigma_{\hat{\theta}}}(M, 1)) = \lim_{\hat{\theta}\to\theta_{\min}} G(\theta_{\sigma_{\hat{\theta}}}(M, 0)) = 0$. Hence,

$$\lim_{\hat{\theta}\to\theta_{\min}} U_S^{\sigma_{\hat{\theta}}}(M, \hat{\theta}) < 0.$$

On the other hand, the cost does not affect the seller's expected payoff from N. Then, since $\lim_{\hat{\theta}\to\theta_{\max}} \theta_{\sigma_{\hat{\theta}}}(M, \omega) = \theta_{\sigma_{MP}}(M, \omega)$,

$$\lim_{\hat{\theta}\to\theta_{\max}} U_S^{\sigma_{\hat{\theta}}}(M, \theta) = b[\lambda(\theta)G(\theta_{\sigma_{MP}}(M, 1)) + (1 - \lambda(\theta))G(\theta_{\sigma_{MP}}(M, 0))] - c.$$

On the other hand, $\lim_{\hat{\theta}\to\theta_{\max}} \theta_{\sigma_{\hat{\theta}}}(N, \omega) = \theta_{\max}$. Thus, if condition (i) is satisfied,

$$b\lambda(\theta_{\max}) < b[\lambda(\theta_{\max})G(\theta_{\sigma_{MP}}(M, 1)) + (1 - \lambda(\theta_{\max}))G(\theta_{\sigma_{MP}}(M, 0))] - c.$$

Hence, under condition (i), we have

$$\lim_{\hat{\theta}\to\theta_{\max}} U_S^{\sigma_{\hat{\theta}}}(M, \theta) > \lim_{\hat{\theta}\to\theta_{\max}} U_S^{\sigma_{\hat{\theta}}}(N, \theta).$$

Since $\lim_{\hat{\theta}\to\theta_{\min}} U_S^{\sigma_{\hat{\theta}}}(N, \hat{\theta}) > 0$, by continuity, there exists at least one $\hat{\theta}^{**} \in (\theta_{\min}, \theta_{\max})$ such that $U_S^{\sigma_{\hat{\theta}}}(M, \hat{\theta}^{**}) = U_S^{\sigma_{\hat{\theta}}}(N, \hat{\theta}^{**})$.

Observe that, given a strategy and b, c reduces the expected payoff from M, while c does not affect the expected payoff from N. Hence, it is easy to see that the largest solution of $U_S^{\sigma_{\hat{\theta}}}(M, \hat{\theta}) = U_S^{\sigma_{\hat{\theta}}}(N, \hat{\theta})$ is strictly larger than the largest equilibrium cutoff in $\Gamma(b, 0)$. Then, without loss of generality, let $\hat{\theta}^{**}$ be the largest solution of $U_S^{\sigma_{\hat{\theta}}}(M, \hat{\theta}) = U_S^{\sigma_{\hat{\theta}}}(N, \hat{\theta})$. We claim that $\sigma_{\hat{\theta}^{**}}$ is an equilibrium strategy if condition (ii) holds. To see the claim, note that

$$\begin{aligned}
\frac{U_S^{\sigma_{\hat{\theta}^{**}}}(M, \theta)}{U_S^{\sigma_{\hat{\theta}^{**}}}(N, \theta)} &= \frac{b[\lambda(\theta)G(\theta_{\sigma_{\hat{\theta}^{**}}}(M, 1)) + (1 - \lambda(\theta))G(\theta_{\sigma_{\hat{\theta}^{**}}}(M, 0))] - c}{b\lambda(\theta)G(\theta_{\sigma_{\hat{\theta}^{**}}}(N, 1))} \\
&= \frac{G(\theta_{\sigma_{\hat{\theta}^*(c)}}(M, 1)) + \left[\frac{1}{\lambda(\theta)} - 1\right] G(\theta_{\sigma_{\hat{\theta}^{**}}}(M, 0)) - \frac{1}{\lambda(\theta)}\frac{c}{b}}{G(\theta_{\sigma_{\hat{\theta}^{**}}}(N, 1))} \\
&= \frac{G(\theta_{\sigma_{\hat{\theta}^*(c)}}(M, 1)) + \frac{G(\theta_{\sigma_{\hat{\theta}^{**}}}(M,0)) - \frac{c}{b}}{\lambda(\theta)} - G(\theta_{\sigma_{\hat{\theta}^{**}}}(M, 0))}{G(\theta_{\sigma_{\hat{\theta}^{**}}}(N, 1))}
\end{aligned}$$

Note that since $\hat{\theta}^* < \hat{\theta}^{**}$, $G(\theta_{\sigma_{\hat{\theta}^*}}(M,0)) < G(\theta_{\sigma_{\hat{\theta}^{**}}}(M,0))$. Thus, $c < bG(\theta_{\sigma_{\hat{\theta}^{**}}}(M,0))$ if condition (ii) is satisfied. Then, since $\frac{1}{\lambda(\theta)}$ is strictly decreasing in θ, $\frac{U_S^{\sigma_{\hat{\theta}^{**}}}(M,\theta)}{U_S^{\sigma_{\hat{\theta}^{**}}}(N,\theta)}$ is also strictly decreasing in θ. Hence, when $\frac{U_S^{\sigma_{\hat{\theta}^{**}}}(M,\theta^{**})}{U_S^{\sigma_{\hat{\theta}^{**}}}(N,\theta^{**})} = 1$, $\frac{U_S^{\sigma_{\hat{\theta}^*(c)}}(M,\theta')}{U_S^{\sigma_{\hat{\theta}^*(c)}}(N,\theta')} < (>) 1$ for $\theta' > (<) \hat{\theta}^{**}$. Moreover, since $c < bG(\theta_{\sigma_{\hat{\theta}^{**}}}(M,0))$, $U_S^{\sigma_{\hat{\theta}^{**}}}(M,\theta) > 0$ for any $\theta < \hat{\theta}^{**}$. This establishes the first part of the proposition.

For the second part, note that $G(\theta_{\sigma_{\hat{\theta}}}(M,0)) < G(\theta_{\sigma_{MP}}(M,0))$ for any $\hat{\theta}$. Thus, if $c > bG(\theta_{\sigma_{MP}}(M,0))$, $\frac{U_S^{\sigma_{\hat{\theta}}}(M,\theta)}{U_S^{\sigma_{\hat{\theta}^*}}(N,\theta)}$ is strictly increasing given any solution of $U_S^{\sigma_{\hat{\theta}}}(M,\hat{\theta}) = U_S^{\sigma_{\hat{\theta}}}(N,\hat{\theta})$. Q.E.D.

9.7 Proof of Proposition 7

Suppose there are two sellers and they use a cutoff countersignaling strategy with cutoff $\hat{\theta}$. The probability that seller i's product has the highest value conditional on $(\omega_i, \theta_j, a_i, a_j)$ is

$$\Phi_{a_i,a_j}(\theta_j,\omega_i|\hat{\theta}) = \begin{cases} \begin{array}{c} \lambda(\theta_j)\Pr\left(\theta^i_{\hat{\theta}}(a_i,\omega_i) > \max\{\theta_0, \theta^j_{\hat{\theta}}(M,1)\}\right) \\ +(1-\lambda(\theta_j))\Pr\left(\theta^i_{\hat{\theta}}(a_i,\omega_i) > \max\{\theta_0, \theta^j_{\hat{\theta}}(M,0)\}\right) \end{array} & \text{if } a_j = M \\ \begin{array}{c} \lambda(\theta_j)\Pr\left(\theta^i_{\hat{\theta}}(a_i,\omega_i) > \max\{\theta_0, \theta^j_{\hat{\theta}}(N,1)\}\right) \\ +(1-\lambda(\theta_j))\Pr\left(\theta^i_{\hat{\theta}}(a_i,\omega_i) > \theta_0\right) \end{array} & \text{if } a_j = N \end{cases}$$

Then, seller i's expected payoff from M is

$$\begin{aligned} U_S^{\sigma_{\hat{\theta}}}(M,\theta_i) &= \lambda(\theta_i)\left[\int_{\theta_j<\hat{\theta}} \Phi_{M,M}(\theta_j,1)f(\theta_j)d\theta_j + \int_{\theta_j>\hat{\theta}} \Phi_{M,N}(\theta_j,1)f(\theta_j)d\theta_j\right] \\ &\quad +(1-\lambda(\theta_i))\left[\int_{\theta_j<\hat{\theta}} \Phi_{M,M}(\theta_j,0)f(\theta_j)d\theta_j + \int_{\theta_j>\hat{\theta}} \Phi_{M,N}(\theta_j,0)f(\theta_j)d\theta_j\right] \\ &= \lambda(\theta_i)\int_{\theta_j<\hat{\theta}} \left[\lambda(\theta_j)G(\theta^i_{\hat{\theta}}(M,1))\frac{1}{2} + (1-\lambda(\theta_j))G(\theta^i_{\hat{\theta}}(M,1))\right] f(\theta_j)d\theta_j \\ &\quad +\lambda(\theta_i)\int_{\theta_j>\hat{\theta}} (1-\lambda(\theta_j))G(\theta^i_{\hat{\theta}}(M,1))f(\theta_j)d\theta_j \\ &\quad +(1-\lambda(\theta_i))\int_{\theta_j<\hat{\theta}} (1-\lambda(\theta_j))G(\theta^i_{\hat{\theta}}(M,0))\frac{1}{2}f(\theta_j)d\theta_j \\ &\quad +(1-\lambda(\theta_i))\int_{\theta_j>\hat{\theta}} (1-\lambda(\theta_j))G((\theta^i_{\hat{\theta}}(M,0))f(\theta_j)d\theta_j \end{aligned}$$

On the other hand, seller i's expected payoff from N is

$$\begin{aligned}
U_S^{\sigma_{\hat\theta}}(N,\theta_i) &= \lambda(\theta_i)\left[\int_{\theta_j<\hat\theta}\Phi_{N,M}(\theta_j,1|\hat\theta)f(\theta_j)d\theta_j + \int_{\theta_j>\hat\theta}\Phi_{N,N}(\theta_j,1|\hat\theta)f(\theta_j)d\theta_j\right] \\
&= \lambda(\theta_i)\int_{\theta_j<\hat\theta}\left[\lambda(\theta_j)G(\theta^i_{\hat\theta}(N,1)) + (1-\lambda(\theta_j))G(\theta^i_{\hat\theta}(N,1))\right]f(\theta_j)d\theta_j \\
&\quad +\lambda(\theta_i)\int_{\theta_j>\hat\theta}\left[\lambda(\theta_j)G(\theta^i_{\hat\theta}(N,1))\frac{1}{2} + (1-\lambda(\theta_j))G(\theta^i_{\hat\theta}(N,1))\right]f(\theta_j)d\theta_j.
\end{aligned}$$

First, by analogous reasoning to the single seller case, $\lim_{\hat\theta\to\theta_{\min}} U_S^{\sigma_{\hat\theta}}(M,\hat\theta) = 0$ and $\lim_{\hat\theta\to\theta_{\min}} U_S^{\sigma_{\hat\theta}}(N,\hat\theta) > 0$. Moreover, observe that if $\lambda(\theta)$ is sufficiently small for some θ', $U_S^{\sigma_{\hat\theta}}(M,\theta') > U_S^{\sigma_{\hat\theta}}(N,\theta')$. Then, since $U_S(N,\hat\theta_i|\hat\theta)$ and $U_S(M,\hat\theta_i|\hat\theta)$ are both continuous in $\hat\theta$, there exists at least one $\hat\theta^* \in (\theta_{\min},\theta')$ such that $U_S^{\sigma_{\hat\theta^*}}(N,\hat\theta^*_i) = U_S^{\sigma_{\hat\theta^*}}(M,\hat\theta^*_i)$.

Now, let

$$\begin{aligned}
W_i(M,1) &= \int_{\theta_j<\hat\theta}\Phi_{M,M}(\theta_j,1)f(\theta_j)d\theta_j + \int_{\theta_j>\hat\theta}\Phi_{M,N}(\theta_j,1)f(\theta_j)d\theta_j, \\
W_i(M,0) &= \int_{\theta_j<\hat\theta}\Phi_{M,M}(\theta_j,0)f(\theta_j)d\theta_j + \int_{\theta_j>\hat\theta}\Phi_{M,N}(\theta_j,0)f(\theta_j)d\theta_j, \\
W_i(N,1) &= \int_{\theta_j<\hat\theta}\Phi_{N,M}(\theta_j,1|\hat\theta)f(\theta_j)d\theta_j + \int_{\theta_j>\hat\theta}\Phi_{N,N}(\theta_j,1|\hat\theta)f(\theta_j)d\theta_j.
\end{aligned}$$

Then,

$$\begin{aligned}
\frac{U_S^{\sigma_{\hat\theta^*}}(M,\theta_i)}{U_S^{\sigma_{\hat\theta^*}}(N,\theta_i)} &= \frac{\lambda(\theta_i)W_i(M,1) + (1-\lambda(\theta_i))W_i(M,0)}{\lambda(\theta_i)W_i(N,1)} \\
&= \frac{W_i(M,1) + \frac{1-\lambda(\theta_i)}{\lambda(\theta_i)}W_i(M,0)}{W_i(N,1)}
\end{aligned}$$

Since $\frac{1-\lambda(\theta)}{\lambda(\theta)}$ is strictly decreasing in θ, $\frac{U_S^{\sigma_{\hat\theta^*}}(M,\theta)}{U_S^{\sigma_{\hat\theta^*}}(N,\theta)}$ is also strictly decreasing in θ. Thus, $\frac{U_S^{\sigma_{\hat\theta^*}}(M,\theta')}{U_S^{\sigma_{\hat\theta^*}}(N,\theta')} < (>)1$ for $\theta' > (<)\hat\theta^*$, that is, no type has incentive to deviate from $\sigma_{\hat\theta^*}$.

To show that any informative equilibrium is a cutoff countersignaling equilibrium, suppose there exists θ',θ'' such that $\sigma(\theta') = N$, $\sigma(\theta'') = M$ and $\theta' < \theta''$ in an equilibrium. Then, the following conditions have to be satisfied.

$$\begin{aligned}
\lambda(\theta')W_i(N,1) &\geq \lambda(\theta')W_i(M,1) + (1-\lambda(\theta'))W_i(M,0), \\
\lambda(\theta'')W_i(N,1) &\leq \lambda(\theta'')W_i(M,1) + (1-\lambda(\theta''))W_i(M,0).
\end{aligned}$$

Thus,

$$\frac{\lambda(\theta'')W_i(M,1)+(1-\lambda(\theta''))W_i(M,0)}{\lambda(\theta'')W_i(N,1)} \geq 1 \geq \frac{\lambda(\theta')W_i(M,1)+(1-\lambda(\theta'))W_i(M,0)}{\lambda(\theta')W_i(N,1)}.$$

or

$$\frac{W_i(M,1)+\frac{1-\lambda(\theta'')}{\lambda(\theta'')}W_i(M,0)}{W_i(M,1)} \geq 1 \geq \frac{W_i(M,1)+\frac{1-\lambda(\theta')}{\lambda(\theta')}W_i(M,0)}{W_i(M,1)}.$$

However, since $\frac{1-\lambda(\theta'')}{\lambda(\theta'')} < \frac{1-\lambda(\theta')}{\lambda(\theta')}$, the above inequality cannot be satisfied, a contradiction. Q.E.D.

References

[1] Alba, J.W., Hutchinson, J.W., Lynch,J.G., "Memory and Decision Making," in H. Kassarjian and T. Robertson (Eds.) Handbook of Consumer Behavior. Englewood Cliffs, NJ: Prentice Hall, 1-49. (1991)

[2] Crawford, V., Sobel, J., "Strategic information transmission," *Econometrica* (1982)

[3] Eliaz, K., Spiegler, R., "Consideration sets and competitive marketing," *Review of Economic Studies* (2011)

[4] Eliaz, K., Spiegler, R., "On the strategic use of attention grabbers," *Theoretical Economics* (2011)

[5] Feltovich, N., Harbaugh, R., To, T., "Too Cool for School? Signalling and Countersignalling," *Rand Journal of Economics* (2002)

[6] Grossman, S,. Perry, "Perfect sequential equilibrium," *Journal of Economic Theory* (1991)

[7] Manzini, P., Mariotti, M., "Sequentially Rationalizable Choice." *American Economic Review*, (2007)

[8] Masatlioglu, Y, Nakajima, D., "Choice by Iterative Search." Working paper (2008)

[9] Masatlioglu, Y., Nakajima, D., Ozbay, E., "Revealed attention." *American Economic Review* (2009),

[10] Milgrom, P., Roberts, J., "Price and Advertising Signals of Product Quality," *Journal of Political Economy* (1986)

[11] Mullainathan S, Schwartzstein, J., Shleifer, A, "Coarse Thinking and Persuasion," *Quarterly Journal of Economics* (2008)

[12] Nelson, P., "Advertising as information," *Journal of Political Economy*, (1974)

[13] Shapiro, J. "A 'Memory-Jamming' Theory of Advertising," Mimeo, University of Chicago, (2006)

[14] Shocker, A., Ben-Akiva, M., Boccara, B., Nedungadi, P., "Consideration Set Influences on Consumer Decision-Making and Choice: Issues, Models, and Suggestions," *Marketing Letters* (1991)

[15] Stigler, G., "The economics of information," *Journal of Political Economy*, (1961)