

JENA ECONOMIC RESEARCH PAPERS



2011 – 020

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www.jenecon.de

ISSN 1864-7057

The JENA ECONOMIC RESEARCH PAPERS is a joint publication of the Friedrich Schiller University and the Max Planck Institute of Economics, Jena, Germany. For editorial correspondence please contact markus.pasche@uni-jena.de.

Impressum:

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May 2011

Abstract

Different evaluators typically disagree how to rank different candidates since they care more or less for the various qualities of the candidates. It is assumed that all evaluators submit vector bids assigning a monetary bid for each possible rank order. The rules must specify for all possible vectors of such vector bids the collectively binding rank order of candidates and the "payments" for this bid vector and its implied rank order. Three axioms uniquely define the "procedurally fair" ranking rules. We finally discuss how our approach can be adjusted to situations where one wants to rank only acceptable candidates.

JEL classification: *C70, C72, D63, D71*
Keywords : *social ranking, fairness, fair game forms, objective equality, mechanism design, committee decision making*

1. Introduction

When facing several candidates we often engage in ranking them. Examples are sports contests like the Olympics where the task is easy when there exists some numerically measurable success like distance, speed, weight etc. But there are other sports like gymnastics where different evaluators may rank differently and where collective ranking depending on divergent individual rankings poses quite a challenge. This is also true for competitions in the arts etc. where one does not only care who wins but also for the entire ranking of candidates. Similarly, in national concourses of candidates, e.g. in France or Italy when hiring young professors, one may not only mind who is first but also which ranking of all candidates is the most adequate one.

If evaluators care for the whole collective ranking, it makes sense to incentivize them by making them "pay" for whatever they choose. Here "pay" may also mean to be monetarily compensated when having to accept a collective ranking which one does not like at all. We therefore assume that all evaluators submit a bid for all possible rankings of candidates, i.e., each evaluator i submits a vector bid b_i. The rules have then to determine a collective ranking for all possible vectors of individual vector bids b_i

- the collective ranking and
- the individual "payments" implied by this bid vector and its resulting collective ranking.

Using similar axioms as in Güth (2010) and Güth and Kliemt (2011), we derive the rules managing these two tasks for all possible bid vectors. Due to the non-discriminatory nature of these rules, we refer to them as a procedurally fair mechanism to collectively rank candidates.

Section 2 presents the axioms and derives the procedurally fair mechanism, implied by them. An example of a collective ranking task is presented in section 3. In section 4, we describe how to extend our approach to situations where one endogenously wants to determine which candidates are acceptable and where one collectively ranks only the acceptable candidates. Section 5 concludes.

2. Axioms and derivation of rules

Let $I=\{a,b,c,...\}$ denote the finite set of candidates where we refer by $i\in I$ to an arbitrary member of candidate set I with $2\leq \#I<\infty$. The evaluators are the members of the evaluation committee $N=\{1,...,n\}$ with $n\geq 2$. We denote by Π the set of all linear rankings $\pi=\left(\Pi(i)_{i\in I}\right)$ of candidates with $1\leq \pi(k)\leq \#I; \pi(k)\neq\pi(l)$ for $k,l\in I$ and $k\neq l$ meaning that if, for instance $\pi(a)=2$, candidate a is evaluated as the second best candidate in I. The task is to select one $\pi\in\Pi$ which determines the collectively binding ranking of all candidates.

To let the individual evaluators influence which $\pi\in\Pi$ will be chosen, we assume that each evaluator $j\in N$ submits a vector bid

$b_j=\left(b_j(\pi)_{\pi\in\Pi}\right)$

assigning a monetary bid $b_j(\pi)\in\mathbb{R}$ to all possible rankings $\pi\in\Pi$. We refer to

$b=(b_1,...,b_n)$

as the bid vector, composed of the n vector bids b_j.

The task of the rules is to specify for each bid vector b

- which collective ranking $\pi^*(b)\in\Pi$ is selected and
- which payments $c_j\left(\pi^*(b),b\right)\in\mathbb{R}$ are imposed on the n evaluators $j=1,...,n$.

To uniquely derive these rules, we impose

Axiom O (optimality with respect to bids):

For all bid vectors b the selected collective ranking $\pi^*(b)$ must satisfy

$\sum_j b_j\left(\pi^*(b)\right)\geq\sum_j b_j(\pi)$ for all $\pi\in\Pi$.

Axiom K (non-profitability).

For all bid vectors b "payments" add up to 0, i.e.,

$\sum_j c_j\left(\pi^*(b),b\right)=0$.

Axiom E (equality of net benefits with respect to bids):

For all bid vectors b and $k, j \in N$:

$$b_k\left(\pi^*(b)\right) - c_k\left(\pi^*(b), b\right) = b_j\left(\pi^*(b)\right) - c_j\left(\pi^*(b), b\right).$$

Due to axiom E, we can assume

$b_j\left(\pi^*(b)\right) - c_j\left(\pi^*(b), b\right) = \Delta$ for all $j \in N$.

Aggregating over $j \in N$ thus yields

$$\sum_j b_j\left(\pi^*(b)\right) = n\Delta$$

due to axiom K so that

$$c_j\left(\pi^*(b), b\right) = b_j\left(\pi^*(b)\right) - \frac{\sum_k b_k\left(\pi^*(b)\right)}{n}$$

for all $j \in N$. Altogether this proves

Proposition: For all bid vectors b the procedurally fair mechanism for collectively ranking candidates

- selects the ranking with the maximal sum of bids (axiom O) and
- imposes for all $j \in N$ the payment

$$c_j\left(\pi^*(b), b\right) = b_j\left(\pi^*(b)\right) - \frac{\sum_j b_j\left(\pi^*(b)\right)}{n}.$$

Thus, one maximizes the sum of bids and equally redistributes this maximal bid sum in order to fulfill requirements K and E.

Note that if the bid sum is negative, evaluators j have to pay more than their bid $b_j\left(\pi^*(b)\right)$ whereas they pay less than their bid when this bid sum is positive. It is, of course, necessary that these payments are compulsory, i.e., being an evaluator $j \in N$ and having to accept the collective ranking $\pi^*(b)$ and to pay $c_j\left(\pi^*(b), b\right)$ is an exogenously given obligation which one cannot avoid. What we thus consider are evaluation committees to which one naturally belongs either by birth, citizenship, or other given and unchangeable qualities.

What we have derived is – in game theoretic terminology – a game form and not a game proper. The latter would have to be introduced when trying to run a test bed

experiment to explore the behavioral reliability of the procedurally fair mechanism for collectively ranking candidates.

Of course, there are many ways to enrich the game form such that it becomes a game proper. But this is typical for procedurally fair mechanism design. It only derives the game form as in legal or constitutional design which then, given the specific circumstances of a given case, provides the basis for a game theoretic analysis, e.g. by including the players' – here the evaluators – true evaluations of collective rankings $\pi \in \Pi$ as well as the beliefs concerning the true evaluations of other evaluators.

3. An example

As an example we use the experimental scenario of Güth, Levati, and Montinari (2011) who study whether a procedurally fair mechanism, as derived above, crowds out other regarding concerns, respectively crowds in opportunistic self-serving. Following Güth and Selten (1991), the three-person voting paradox (Condorcet, 1785) is cardinalized by letting the three voters 1,2,3 evaluate their best (worst) of the three alternative candidates A, B, C by 1(0) whereas the median one of 1,2,3 is evaluated by a, b, and c, respectively, where $0 < a, b, c < 1$. In cardinal utility terms, we rely on

$$u_1(B) = 1 > u_1(A) = a > u_1(C) = 0,$$

$$u_2(C) = 1 > u_2(B) = b > u_2(A) = 0,$$

$$u_3(A) = 1 > u_3(C) = c > u_3(B) = 0.$$

Thus, the majority of 1 and 3, 2 and 3, and 1 and 2 prefers A over C, C over B, and B over A illustrating the intransitive majority ranking. To derive from this evaluations of collective rankings $\pi = (\pi_1, \pi_2, \pi_3)$ with $\pi_i, \pi_j \in \{A, B, C\}$ and $\pi_i \neq \pi_j$, Güth, Levati, and Montinari (2011) simply weight π_1 by ½, π_2 by 1/3, and π_3 by 1/6 implying that for $i = 1,2,3$ the most preferred ranking is evaluated with $1/2 + a/3, 1/2 + b/3$ and $1/2 + c/3$, respectively, whereas $\pi = (A, B, C)$, for instance, is evaluated with $\frac{a}{2} + \frac{1}{3}$ by 1, with $\frac{b}{3} + \frac{1}{6}$ by 2, and with $\frac{1}{2} + \frac{c}{6}$ by 3.

4. Restriction to acceptable candidate sets

Let A with $\varnothing \neq A \subseteq I$ denote an arbitrary, non-empty subset of candidates, Π_A the set of all possible rankings $\pi_a \in \Pi_a$ of only the candidates in A. To check whether a set A of candidates is acceptable, we assume vector bids

$b_j = \left(b_j(\pi_a)\right)_{\pi_a \in \Pi_A} \left| \varnothing \neq A \subseteq I \right\rangle$

asking each evaluator $j \in N$ to bid not only for all rankings $\pi \in \Pi$ for $A = I$ but also for all rankings of all non-empty subsets $A \subseteq I$. We denote again by b the vector of such individual vector bids b_j.

With the help of this notation, we can define acceptable candidate sets A with $\varnothing \neq A \subseteq I$ by postulating that such sets must satisfy

$\sum_j b_j(\pi_a) \geq 0$ for all $\pi_a \in \Pi_a$.

Axiom A (acceptability with respect to bids):

> If all A with $\varnothing \neq A \subseteq I$ are unacceptable, then the only acceptable candidate set is the empty set so that there is no need of collective ranking and the "payments" $c_j(b)$ for all $j \in N$ are set equal to 0.
>
> Otherwise the collective ranking task is restricted to acceptable candidate sets A with $\varnothing \neq A \subseteq I$.

All what is needed in addition to axiom A is to adapt our previous three axioms O, K, and E.

Axiom O' (optimality with respect to bids):

> For all bid vectors b with a non-empty set $\mathrm{A}(b)$ of acceptable candidate sets A with $\varnothing \neq A \subseteq I$, the selected collective ranking $\pi_a^*(b) \in \Pi\left(A^*(b)\right)$ must satisfy
>
> $\sum_j b_j\left(\pi_a^*(b)\right) \geq \sum_j b_j(\pi_a)$ for all $\pi_a \in \Pi_A$ and all $A \in \mathrm{A}(b)$.

Axiom K' (non-profitability).

For all bid vectors b with $\mathrm{A}(b) \neq \varnothing$, "payments" add up to 0, i.e.,

$\sum_j c_j\left(\pi_a^*(b), b\right) = 0$.

Axiom E' (equality of net benefits with respect to bids):

For all bid vectors b with $\mathrm{A}(b) \neq \varnothing$ and $k, j \in N$:

$b_k\left(\pi_a^*(b)\right) - c_k\left(\pi_a^*(b), b\right) = b_j\left(\pi^*(b)\right) - c_j\left(\pi_a^*(b), b\right)$.

Proceeding as in the previous section obviously yields

$$c_j\left(\pi_a^*(b), b\right) = b_j\left(\pi_a^*(b)\right) - \frac{\sum_k b_k\left(\pi_a^*(b)\right)}{n} \text{ for all } j \in N$$

and all bid vectors b with $\mathrm{A}(b) \neq \varnothing$ what demonstrates how our analysis can be extended to situations where the acceptability of the candidate set is endogenously determined by the evaluators $j \in N$.

5. Conclusions

We consider situations where a group N of evaluators, who can be individuals or groups of individuals, has to collectively rank several candidates which can be physical objects like pieces of art, individuals, e.g. candidates for a job, or teams of individuals like sports teams. Our basic approach assumes that one can "incentivize" evaluators by asking them to determine vector bids for all possible rankings and to "pay". To derive procedurally fair rules for selecting the collective ranking and the individual "payments", we impose some – in our view – quite acceptable requirements defining a "procedurally fair" game form for such tasks.

Of course, such procedurally fair mechanisms do not qualify as revelation mechanisms. Quite to the contrary, when inducing true individual evaluations of collective rankings and beliefs concerning others' true individual evaluations of collective rankings, the games proper will usually offer incentives for strategic misrepresentation. In our view, mechanism design as, for instance, legal and institution design does not try to deal with specific situations allowing to model them

as games proper but with classes of situations, possibly infinitely many, for which one can provide a common game form but not an all comprising game proper.

When deriving such game form where true evaluations and beliefs concerning them are missing, what we suggest is to “regulate with respect to bids”, i.e., by conditioning on objectively observable numerical variables. It is this aspect where our approach is based on earlier contributions (Güth, 1986; Güth and Van Damme, 1986; Güth and Peleg, 1996, Güth, forthcoming; Güth and Kliemt, 2011) which also “regulate with respect to bids”. Whereas the earlier studies are postulating envy-free net trades according to bids, the latter ones are mainly based on “equality of net gains according to bids” which is also the main axiom here. In our view, the equal treatment of evaluators in view of their bids, i.e., of their objectively observable and even measurable value statements, is an obvious and rather convincing requirement of procedural fairness.

Thus, the crucial feature of our approach which will often question its practical applicability is that we assume that evaluators can be made to “pay” in the sense of monetarily contributing or of monetary compensations. We do not claim that this is always convincing but that at least for some situations this offers an innovative way to overcome the difficulties when groups of evaluators have to agree on collective rankings of several candidates.

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