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Endogenous Money – On Banking Behaviour in New and Post Keynesian Models

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Abstract

In New and Post Keynesian macroeconomic models, money supply is assumed to be endogenous. In this paper we explicitly derive the behaviour of the banking sector regarding the loan supply, bonds demand, and demand for reserves from portfolio and liquidity considerations. The process of money creation is determined by the interaction of banks, non-banks, and the central bank on the interdependend markets for reserves, loans, and bonds. Although the microeconomics of bank behaviour is modelled quite simply, interest rates as well as monetary aggregates depend on policy variables in a non-linear and non-monotonous way.

Keywords: endogenous money, loans market, bonds market, central banking

JEL Classification: E51, E44, B22

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1 Introduction

The endogeneity of money supply is a widely discussed topic, especially in New and Post Keynesian macroeconomics. It can be taken as a common conviction that individual behaviour regarding credit demand and supply as well as holding currency and deposits has an impact on the money creation process. These issues are often neglected in Neoclassical and Monetarist type models. There are, however, very different approaches how endogeneity of money originates (for an extensive review see e.g. Palley (2002), Palley (2008*b*)). New Keynesian economics (see e.g. Mankiw and Romer (1991), Romer (2000), Woodford (2003)) is dominated by the *"New Consensus"* where the exogenously determined money supply of the central bank (LM curve) is replaced by the Taylor rule (Taylor (1993)). The monetary policy targets inflation and output gap by controlling the real interest rate, while there is no explicit theory about the creation of credit and money.

In Post Keynesian economics money is endogenous by its nature (Lavoie (1992), Lavoie (2006), Rochon (1999)). There have been two distinct approaches developed which are usually denoted as the *"accomodationist"* (or horizontalist) and the *"structuralist"* (or verticalist) approach (see Moore (1988), Pollin (1991), Fontana (2004), Wray (2007), Palley (2008*a*)). Both schools have in common that the money creation process is determined by the behaviour of commercial banks and non-banks on the credit market. The process starts with credit demand, and credit creates deposits. The accomodation approach argues that an increase in credit demand leads to a need for additional reserves. In order to ensure the liquidity of the banking sector the central bank has to respond by increasing the money base and hence to accomodate the credit demand. In this view the microeconomic considerations of the commercial banking sector play a minor role. In contrast, the structuralist approach argues that commercial banks respond to an increase in credit demand with structural changes of their portfolio on the asset and the liability side. This may lead to a change in the demand for reserves and hence in the interaction with the central bank. However, there is no monotone relationship between credit demand and the response of the central bank, but complex structural effects on the interest rates and portfolio composition. While the accomodationists see the central bank's behaviour as a reflex to the non-bank public (which hence determines solely the money supply), the structuralists see a certain degree of autonomous central banking policy, hence money is endogenously generated by the interaction of the public, the central bank, and the commercial banks.

We argue, that it is important to investigate these complex interactions to understand how central bank policy impulses are conducted to the real sphere via credit and bonds market, as well as to understand how the real sphere affects the money creation process. Therefore, our approach is related to the structuralist view. We will not consider any strategic policy implications like the Taylor rule since such rules make sense only in the context of a full-fledged macroeconomic model. It is important to understand how commercial banks behave on credit, bond and reserve markets, and how they respond to changes on these markets as well as to changes in the central bank policy. As we will see, open market operations and changes in interest rates for borrowed reserves have – depending on the parametrization – complex and sometimes countervailing effects on variables like credit supply or bonds demand. Therefore, it is more reasonable to combine such a building block of the financial sector, as outlined in this paper, with a complete macroeconomic model, and then – if possible – to derive a rationale for monetary policy rules.

In contrast to most Post Keynesians who assume simple markup pricing for determining the loans interest rate, we develop a model of banking behaviour which is in some sense neoclassical: the (representative) bank has preferences regarding risk, return, and liquidity, and it manages its assets and liabilities via portfolio and Value at Risk techniques. A single commercial bank operates in competitive markets and responds to changes in market conditions as well as to changes in central bank policy. This allows for a detailed analysis of some spillover effects between credit and bonds market, the market for reserves, and the real sector (via income). Although the microeconomics of banking are portrayed in a very simplified way the results are not trivial.

The paper is organized as follows: Before developing our model, we briefly discuss two sources of endogeneity by means of two approaches in the literature. In section 2 we review the model of Bernanke and Blinder (1988) who introduce the idea how portfolio considerations of the commercial bank affect the money multiplier. Section 3 discusses the less common approach of Bofinger (2001) where changes on the credit market affects the bank's demand for central bank loans. This establishes a close relation between the interest rates on the market for credits and the market for central bank money via an optimization calculus of the commercial bank. Section 4 picks up both ideas in a consistent framework and extends them with liquidity considerations. These liquidity issues are twofold: When the bank's capital is fixed the volume of risky assets has to be restricted. On the liability side there is a risk of unperceived outflows of deposits which requires to hold a sufficient volume of liquid assets like excess reserves. All portfolio and Value at Risk decisions are calculated explicitly and exhibit nonlinear relationships between the central variables (e.g. loans (bonds) demand (supply)) and the interest rates. The money creation process is analysed in section 5. I starts with some multiplier considerations, and then develops a model of the financial sector which includes also non-bank behavior and discusses the the interdependency with the real sector of the economy. Section 6 gives a numerical example, section 7 concludes.

2 The approach by Bernanke and Blinder

In the approach by Bernanke and Blinder (1988), the commercial bank's simplified balance sheet contains reserves (R), loans (L^s), and bonds (B^b) as assets, while deposits (D) are the unique liability. There are no currencies and no central bank loans to commercial banks. The reserve requirements are rD, hence the balance sheet can be written as $E + L^s + B^b = (1-r)D$, where E are the excess reserves at the central bank with a zero interest rate. Since loans and bonds have both expected returns and a certain risk (credit failure and bonds price volatility) the commercial bank has portfolio considerations about its assets. The structure of the portfolio is given by:

$$\begin{aligned} E(i) &= \lambda_E(i)(1-r)D \\ L^s(i,\rho) &= \lambda_L(i,\rho)(1-r)D \\ B^b(i,\rho) &= (1-\lambda_E(i)-\lambda_L(i,\rho))(1-r)D \end{aligned} \tag{1}$$

where i is the interest rate of the bonds, and ρ is the interest rate of loans. Obviously λ_L depends positively on ρ, negatively on i, and vice versa for λ_B. For simplicity, Bernanke and Blinder assume that variations in ρ only affect the shares of L^s and B^b in the portolio.

The balance sheet of the central bank is given by

$$R = rD + E = rD + \lambda_E(i)(1-r)D = (r + \lambda_E(i)(1-r))D \tag{2}$$

Hence the money multiplier is $m(i) = [r + \lambda_E(i)(1-r)]^{-1}$. In contrast to the exogenous multipliers in common textbook models there is now a dependency of the multiplier on the behaviour of the commercial bank, i.e. the multiplier depends on the endogenously determined bonds interest rate i.

The equilibrium in the loans market is determined by $L^d(i, \rho, y) = L^s = \lambda_L(\rho, i)(1-r)D$. The demand for loans depends positively on i and income y, and negatively on ρ. The bonds market is not explicitely modelled in the Bernanke/Blinder approach. While the loans and the bonds market determine the money supply $D^s = m(i)R$, the money demand $D^d = D^d(i, y)$ follows the standard assumptions (positive dependency on y and negative dependency on the bonds interest rate i). Money market equilibrium is given by $D^d(i, y) = m(i)R$ which is the conventional LM curve. From these results Bernanke and Blinder construct a so-called CC curve as a substitute for the IS curve where the goods and credit markets are in equilibrium. Together with the LM curve they study the impact of monetary impulses on the real sector.

For the purpose of our paper we are not interested into the CC-LM macro model but we pick up the idea that the commercial bank's behaviour is driven by portfolio considerations, which have important implications for the loans market and the money market. The mechanistic exogenous money multiplier is modified to an endogenous money multiplier, based on the behaviour in the loans market and on portfolio considerations of the commercial bank. There are, however, some shortcomings which deserve an extension of the framework (for further critical remarks see Bajec and Graf Lambsdorff (2006)).

First, there are no central bank loans to the commercial bank, even though the interest rate policy plays a prominent role in central banking. Changes in the central banks interest rate ρ_c for refinancing commercial banks is an important component of monetary policy. If we allow central bank credits L_c with interest rate ρ_c, the commercial bank has not only to decide on the portfolio structure of a given volume, but also on the volume itself.

Second, the bonds market is not modelled explicitely. Bernanke and Blinder implicitly assume that the non-bank's demand for bonds is a residual from net financial wealth plus loans demand minus desired deposits (see Bajec and Graf Lambsdorff (2006), p.10). Since firms and households face budget constraints it is more reasonable to argue that they decide on the desired structure of financial assets like deposits and bonds, and then decide on the volume of the assets, financed also by loans. Thus, the loans demand L^d is not properly derived. Furthermore, if we assume that open market operations are conducted by buying or selling bonds, this also affects the bonds interest rate i and henceforth the portfolio decisions of non-banks.

Third, the bank's portfolio considerations are reduced to risk and return decisions. However, banking management also addresses solvency and liquidity issues. These shape the loan supply, bonds demand, and the extent of refinancing the operations with central bank loans.

Fourth, there are neither domestic nor foreign currencies, and there is no market for equities and derivative financial contracts. It is clear that a model cannot include too many items without loosing the ability to derive clear analytical results. For the sake of simplicity it is admissible to neglect these things. However, the market for financial contracts apart from loans and bonds becomes of growing importance as the recent financial crisis indicates. Since the observed fragility of the inter-related markets challenges monetary policy, it may be worth to include them in the framework.

Fifth, all markets are assumed to be perfect. Starting from Stiglitz and Weiss (1981) there is a broad literature on credit rationing based on asymmetric information which plays a role also in equity markets (see e.g. Hellmann and Stiglitz (2000)). From Neo Keynesian theory we know that rationing changes the agent's calculus. They will adapt their plans so that e.g. rationing on the loans market probably has spillovers to other financial markets as well as to the real sphere. As a result, the macroeconomic effective demand may depend on rationing effects.

It is not possible, of course, to address all mentioned shortcomings. This paper concentrates on the first three mentioned issues. Regarding the central bank loans for commercial banks we first summarize a model by Bofinger (2001).

3 The approach by Bofinger

In Bofinger (2001) (pp. 53) a model of the macroeconomic loans market is presented, where commercial banks are able to refinance their credit supply by central bank loans L_c, i.e. by the demand for reserves. The aim of the model is to explain the money creation process endogenously by the interaction of the market for loans and the market for reserves. There are no bonds, no excess reserves, and we neglect currency in this model. Hence, the simplified commercial bank's balance sheet is $R + L^s = L_c + D$. The loan supply is explicetly derived from a profit maximizing calculus:

$$\max_{L^s} \ \pi = \rho L^s - \rho_c L_c - \beta (L^s)^2 \tag{3}$$

where the term $\beta(L^s)^2$ describes the increasing risk of debt failures. This can be justified by assuming that with an expanding loan volume, the bank finances more and more risky projects, or more debitors have limited soundness. However, it is more reasonable to assume that the debt failure probability depends on ρ rather than L. Since a central bank loan L_c extends the balance sheet of the bank and increases the reserves, the credit expansion follows the multiplier process. When $R = rD$ is subtracted from the balance sheet we have:

$$L^s = L_c + (1-r)D = L_c + (1-r)mL_c \tag{4}$$

Because for the money multiplier $m = 1/r$ holds true in absence of currency and excess reserves, a simple rearrangement leads to $L^s = mL_c$. Substituting $L_c = m^{-1}L^s$ into the profit function the first order condition yields the supply function:

$$L^s = L^s(\rho, \rho_c, \beta) = \frac{1}{2\beta}(\rho - \rho_c/m) \tag{5}$$

which is increasing in ρ. The demand for loans is given by $L^d(\rho, y)$. From the market equilibrium condition $L^d = L^s$ we obtain an equilibrium interest rate $\rho^*(\rho_c, \cdot)$. Of course ρ^* also depends on demand parameters. The loans market equilibrium implies a profit maximizing demand for reserves, i.e. central bank loans. Substituting $L^s = mL_c$ into the

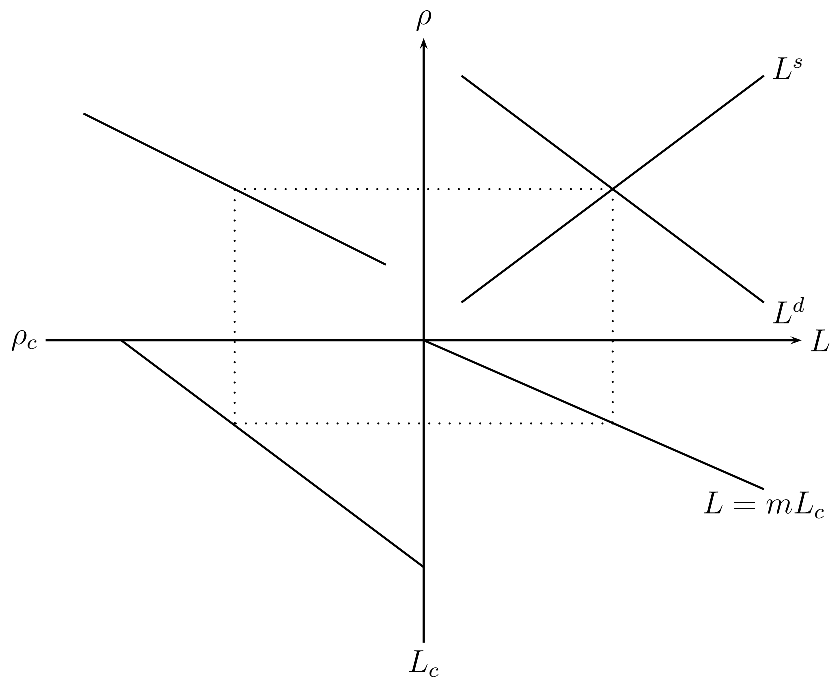

Figure 1: *The Bofinger model as described in Bofinger (2001).*

profit function and maximizing with respect to L_c yields the reserve demand function:

$$L_c(\rho, \rho_c, \beta) = \frac{1}{2m\beta}(\rho - \rho_c/m) \tag{6}$$

On the market for reserves the central bank acts as a monopolist. The central bank chooses a point on the demand function L_c^d according to monetary policy goals instead of profit maximization. A change in the interest rate ρ_c for reserves and therefore a shift of the reserve demand changes the loan supply curve L^s and has therefore an impact on the loan interest rate $\rho^*(\rho_c, \cdot)$.

Assuming a linear loan demand $L^d = \gamma y - \alpha\rho$ it is an easy task to derive the resulting equilibrium interest rates. Figure 1 shows the complete model for the linear case. The upper right quadrant depicts the loans market, the lower right quadrant shows the money multiplier. The loans market equilibrium hence determines the demand for reserves (lower left quadrant) via the multiplier. The relation between the interest rates for loans and reserves ($\rho^*(\rho_c, \cdot)$) is depicted in the upper left quadrant. The interest rate based transmission of monetary impulses works as usual: An increasing ρ_c shifts the loan supply curve upwards. This results in a raise of the market intrerest rate ρ (depending on supply and demand elasticities), and a decrease of the demand for central bank money L_c. However, the money supply is no longer a policy variable, the money (credit) creation process is also determined by the loans demand.

Obviously, the Bofinger model has some shortcomings. As in the Bernanke/Blinder approach, there are no equity markets, no currencies, and no rationing effects due to market imperfections. Moreover, there is neither a bonds market nor excess reserves. So the commercial bank has no asset portfolio and therefore no portfolio considerations (which implies risk aversion while the profit function in the Bofinger model implies risk neutrality). As a consequence, the multiplier is constant. Nevertheless, the money creation process is determined by the behaviour of the commercial banks and the debitors.

Our aim is now to combine Bofinger's idea of an analytically derived demand for reserves, depending on the interest rate policy of the central bank with the Bernanke/Blinder approach which includes a bonds market and portfolio considerations of a (risk averse) commercial bank. Furthermore we extend the framework by liquidity and solvency considerations.

4 An aggregated model of banking behaviour

Starting from the balance sheet of the aggregated banking sector, we derive the decision of a single representative commercial bank regarding the structure and volume of its portfolio. These decisions are driven by considerations about risk, return and liquidity.

4.1 The balance sheet of the aggregated commercial banking sector

The balance sheet of a commercial bank contains three liabilities: deposits D, central bank loans L_c, and bank capital $\overline{BC}$, and the three assets: loans L, bonds B and excess reserves E. The required reserves rD are subtracted from both sides of the sheet. The balance sheet of the commercial bank thus reads:

$$L + B + E = (1 - r)D + L_c + \overline{BC} \tag{7}$$

where the bank capital is assumed to be fixed for simplicity. Since we look at the aggregated banking sector, all inter-bank loans are subtracted from the sheet. Therefore, the market for reserves consists of the aggregated banking sector on the demand side and the central bank on the supply side. All types of reserves demanded by commercial banks which have to pay interest rates to the central bank are subsumed to L_c.

The portfolio considerations of the commercial bank are now twofold. First, the bank has to decide about the portfolio *structure*, which is determined by the shares λ_L, λ_B and λ_E which give the fractions of L, B and E in the full portfolio (with $\lambda_L + \lambda_B + \lambda_E = 1$). The second decision a commercial bank has to make is about the *volume* of the portfolio. The portfolio volume is defined by:

$$V = (1 - r)D + L_c + \overline{BC}. \tag{8}$$

A single bank is seen as not being able to determine the outcome of the macroeconomic deposit creation process. Although an additional credit creates additional deposits, the volume of deposits of a single bank is determined by a process of inflows and outflows of deposits due to the payment behavior of non-banks which cannot be controlled by the beank's behavior. Hence, in a competitive market the single bank will take D as given. Therefore the volume of the portfolio is determined solely by L_c which hence is a policy variable of the bank.

4.2 Management of risk and return

From the three assets L, B and E, there is one riskless asset E with expected return $\mu_E = 0$, and two risky assets L and B. For L, the expected return per unit and the variance are

$$\begin{aligned} \mu_L &= p\rho - (1 - p) \qquad (9) \\ \sigma_L^2 &= p(\rho - \mu_L)^2 + (1 - p)(-1 - \mu_L)^2 \end{aligned}$$

where p is the probability for a successfully returned credit and ρ is the loans interest rate, collaterals have been neglected. As Stiglitz and Weiss (1981) argue, the probability $(1 - p)$ of a complete credit failure may be assumed to be a positive function of ρ. In this paper, however, we take p as exogenously given. The expected return and variance for bonds is

$$\begin{aligned} \mu_B &= i \qquad (10) \\ \sigma_B^2 &= const \end{aligned}$$

For the sake of simplicity we assume that covariances are not present.

The optimal portfolio structure for one riskless and two risky assets is determined in two steps (for details see Huang and Litzenberger (1988)). In the first step, the efficient portfolio frontier for a mix of the two risky assets has to be derived. The risky portfolio R is given by the shares $\tilde{\lambda}_L$ and $\tilde{\lambda}_B = (1 - \tilde{\lambda}_L)$ which implies:

$$\begin{aligned} \mu_R &= \tilde{\lambda}_L \mu_L + \tilde{\lambda}_B \mu_B = \tilde{\lambda}_L (p\rho - (1-p)) + (1 - \tilde{\lambda}_L) i \qquad (11) \\ \sigma_R^2 &= \tilde{\lambda}_L^2 \sigma_L^2 + \tilde{\lambda}_B^2 \sigma_B^2 = \tilde{\lambda}_L^2 (p^2(\rho - \mu_L)^2 + (1-p)^2(-1-\mu_L)^2) + (1-\tilde{\lambda}_L)^2 \sigma_B^2 \end{aligned}$$

Hence, $\tilde{\lambda}_L$ determines all possible (μ_R, σ_R)-combinations, which define the portfolio frontier. In order to find the optimal risky portfolio, which is then mixed with the riskless asset E, we have to determine the tangential point of the efficient portfolio frontier with the capital allocation line (CAL) being defined as:

$$\mu_P = \mu_E + \left(\frac{\mu_p - \mu_E}{\sigma_R} \right) \sigma_P = \left(\frac{\mu_R}{\sigma_R} \right) \sigma_P \qquad (12)$$

Standard portfolio methods provide the solution:

$$\tilde{\lambda}_L = \frac{\mu_L \sigma_B}{\mu_L \sigma_B + \mu_B \sigma_L} \quad \text{and} \quad \tilde{\lambda}_B = 1 - \tilde{\lambda}_L = \frac{\mu_B \sigma_L}{\mu_B \sigma_L + \mu_L \sigma_B} \qquad (13)$$

as there is no covariance present. As μ_L may become negative due to total debt failure, $\tilde{\lambda}_L$ has to be truncated at zero. In the second step, the bank decides how to mix the riskless asset E with the risky portfolio R according to its preferences. To determine the optimal proportion λ_R, the bank maximizes its utility function. We assume a Power function with constant relative risk aversion since this function allows for a separate determination of optimal portfolio structure and optimal portfolio volume, i.e. λ_R is independent from V. The realized value of the portfolio after the investment period is $\tilde{V} = V(1+\lambda_R r)$ with r as the realized return and $E[r] = \mu_R, Var[r] = \sigma_R^2$. The agent subjectively expects a mean wealth $E[\tilde{V}] = V(1 + \mu_P)$ and a variance $Var[\tilde{V}] = V^2 \sigma_P^2$. The Power utility function is $u(\tilde{V}) = \tilde{V}^{(1-\theta)}/(1-\theta)$ where $\theta > 0$ is the constant Arrow-Pratt measure for relative risk aversion. Maximizing $E[u(\tilde{V})]$ with respect to λ_R leads to the well established result from portfolio theory (see appendix):

$$\lambda_R = \min \left\{ \frac{\mu_R}{\theta \sigma_R^2}, 1 \right\} \qquad (14)$$

Note, that λ_R changes as soon as additional constraints from Value at Risk are introduced. Now the bank's optimal portfolio structure is completely determined by:

$$\lambda_L = \tilde{\lambda}_L \lambda_R, \quad \lambda_B = (1 - \tilde{\lambda}_L)\lambda_R, \quad \lambda_E = 1 - \lambda_R \tag{15}$$

where the explicit form is not further revealing and thus have been omitted here.

The next task is to determine the optimal portfolio volume V which is given by (8). By assumption V could be adapted solely by changing reserve demand L_c. Optimality requires that the portfolio volume is expanded by L_c until the expected marginal utility equals the marginal cost ρ_c. In this case it is neccessary to interpret $u(\cdot)$ as a cardinal utility function. To obtain numerically reasonable results, the marginal utility has to be scaled with a scaling parameter $\xi > 0$. For our purposes we set $\xi = 1$, i.e. the scaling is omitted. The calculus $\max_{L_c \geq 0} E[u(\tilde{V})] - \rho_c L_c$ leads to

$$L_c = \left(\frac{(1 + \lambda_R \mu_R - \frac{1}{2}\theta \lambda_R^2 \sigma_R^2)^{(1-\theta)}}{\rho_c} \right)^{1/\theta} - (1-r)D - \overline{BC} \tag{16}$$

Observe, that the effect of an increasing portfolio performance μ_R on L_c is negative (positive) for $\theta > 1$ ($\theta < 1$) as it can seen by differentiating (16) with respect to μ_R. Increasing portfolio performance leads to higher *total* expected utility, accelerated by an increase of λ_R, but this implies a lower *marginal* utility due to the concavity of $u(\cdot)$. Hence the portfolio will be sized down. Only in case of low risk aversion ($\theta < 1$) the marginal utility of the last portfolio unit and therefore borrowed reserves increase. Inserting (14) for λ_R and (11) for μ_R and σ_R^2 into (16), the demand for central bank loans L_c is given by:

$$L_c = L_c(\rho, i, \underset{-}{\rho_c}, \underset{-}{D}) \tag{17}$$

which determines the volume V.

With the optimal shares λ_L, λ_B and λ_E and the optimal portfolio volume V, we obtain the demand for bonds B, the excess reserves E, and the supply of loans L:

$$L = \lambda_L V, \quad B = \lambda_B V, \quad E = \lambda_E V \tag{18}$$

Again, the explicit form is omitted here. For two reasons the dependencies on ρ and i may be not clear: First, since σ_L^2 depends on ρ, an increase in ρ may have ambiguous effects on

λ_R, the share of the risky assets. Second, the marginal utility of the portfolio changes with increasing i or ρ which may have a negative effect on L_c and hence L and B (depending on θ). Then the shares λ_L, λ_B and L_c may have different signs in their derivatives with respect to i and ρ. The structural and the volume effects may be countervailing, thus the total effect depends on the parametrization.

4.3 Mangement of liquidity by Value at Risk (VaR)

In the last section the commercial bank's goal was to balance risk and expected returns. But banks are also interested to keep a certain level of capital in order to stay solvent. Loans may fail and the bonds position in the portfolio is also volatile. Only the excess reserves E are risk-free. Depending on the probability distributions of μ_L, μ_B and the optimal shares λ_L, λ_B it is possible to derive a probability distribution for the losses of the portfolio. With a certain probability the losses could exceed the bank's capital $\overline{BC}$. In this case the bank would be insolvent.

We assume that the bank's management addresses this problem with the Value at Risk (VaR) approach (for details see e.g. Wahl and Broll (2003)). Let α be the probability that the losses exceed the bank's capital, then

$$\text{VaR}_\alpha = q_\alpha V \tag{19}$$

determines the capital requirement to ensure solvency with probability $1 - \alpha$ in a given period. Here V is the portfolio volume and $-q_\alpha$ is α-fractile of the probability distribution (see Figure (2)).

The VaR approach requires that the capital $\overline{BC}$ covers at least the VaR at the level α, i.e. $\overline{BC} \geq \text{VaR}_\alpha = q_\alpha V$. The more risky the portfolio and the higher the desired probability $(1 - \alpha)$ of staying solvent – either determined by the bank's management or by bank regulation policy – the more capital $\overline{BC}$ is required. It can be shown that, for a given VaR_α, the bank chooses an optimal structure of V and BC. In our approach, however, we take $\overline{BC}$ as a given constant. Hence, VaR_α is a constraint for the portfolio volume V,

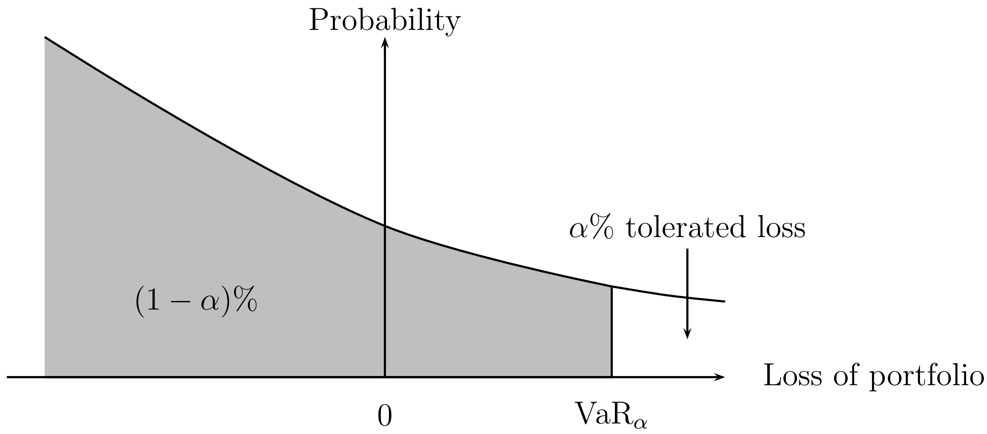

Figure 2: *Loss distribution with Value at Risk.*

leading to:

$$\begin{aligned} \overline{BC} &\geq \text{VaR}_\alpha = q_\alpha V = q_\alpha((1-r)D + L_c + \overline{BC}) \\ \Rightarrow \quad L_c &\leq \frac{1-q_\alpha}{q_\alpha}\overline{BC} - (1-r)D \end{aligned} \tag{20}$$

This is an additional restriction for determining the optimal portfolio volume via L_c as discussed in the previous section. In case that the restriction is binding, the equality sign holds true in (20) and the marginal utility of the portfolio exceeds the marginal cost ρ_c. As a consequence, the money creation process is eventually limited by the bank's solvency policy.

The liquidity management also affects the portfolio structure. If we consider the deposits D not to be a given deterministic value, but a stochastic variable with a given distribution (with D as the expected value), the bank faces the risk of deposit volatility and sudden deposit outflows (bank runs). If depositors wish to draw their deposits, the bank needs immediately liquid assets. We assume that only excess reserves E have the required liquidity (see e.g. Mishkin (2006), pp. 208). Then the VaR approach also applies to the probability to be a victim of bank runs, i.e. to become illiquid. Let β be the probability that sudden outflows of deposists exceed the excess reserves E. Then the bank avoids illiquidity with probability $1-\beta$ if for excess reserves

$$\begin{aligned} E &\geq \text{VaR}_\beta = q_\beta D \\ \Rightarrow \quad \lambda_E &\geq q_\beta \frac{D}{(1-r)D + L_c + \overline{BC}} \equiv \underline{\lambda}_E \end{aligned} \tag{21}$$

holds true. Again, we have an additional constraint for the portfolio calculus as discussed in the previous section. The structure of the risky portfolio $\tilde{\lambda}_L, \tilde{\lambda}_B$ is obviously not affected by the VaR approach. But if (21) is binding then we have $\lambda_R = 1 - \underline{\lambda}_E$ which determines λ_L and λ_B.

Summing up, the VaR approach can be used on the one hand to ensure solvency by balancing V and $\overline{BC}$. This eventually has an impact on the chosen portfolio volume. On the other hand the approach is used to avoid illiquidity in case of deposit outflows by balancing E and D. This eventually has an impact on the chosen portfolio structure. In this paper we do not investigate these effects analytically.

5 Endogenous money supply

5.1 Some multiplier considerations

Up to now the paper addresses only the behavior of commercial banks. Therefore we could only analyse their influence on the endogenous money creation process. It is clear, however, that the behavior of non-banks regarding the credit demand L^d, the bonds demand, and the demand for money (deposits D) play a crucial role because this directly or indirectly affects the bank's decisions about portfolio structure and reserve demand. As we will take D as well as the interest rates on credit and bonds market as given, the analysis must be incomplete. Nevertheless, we are able to highlight the behavior of banks as a hinge between central bank policy and the financial markets. In a framework of endogenous money, monetary policy is about how the central bank *influences* the interactions on the credit and bonds market rather than *determining* their outcome (see also Chick and Dow (2002)).

The multiplier analysis treats D as an outcome of a mechanistic process. In a first step, we derive a money multiplier under the assumption that i and ρ are given exogenous variables. Let $e = E/D = \lambda_E((1-r)D + L_c + \overline{BC})/D$, then the central bank's balance sheet can be expressed as

$$(MB =) \quad S + L_c = (r + e)D \tag{22}$$

with MB as the money base and S as the securities held by the central bank (e.g. bonds).

It is assumed that S is determined by purchases and sales of the central bank on the market for securities. Note again, that in this paper we neglect any currency. The ratio e depends on D and on endogenously determined values of λ_E and L_c. Inserting e into (22) and solving for D we obtain:

$$D = \frac{S + (1-\lambda_E)L_c - \lambda_E \overline{BC}}{r + (1-r)\lambda_E} \tag{23}$$

The central bank could conduct open market operations on the market for securities, which leads c.p. to the multiplier:

$$\left.\frac{dD}{dS}\right|_{L_c=const} = \frac{1}{r + (1-r)\lambda_E} > 0 \tag{24}$$

where λ_E is endogenously determined but is assumed to have a given value throughout the multiplier process. The result is essentially the same as the multiplier derived by Bernanke and Blinder (1988). This is a conventional view of the multiplier process which is slightly enriched by the assumption that the fraction of excess reserves is endogenously determined by portfolio considerations. The multiplier is very sensitive to changes in λ_E. In the 2008 financial crisis most central banks decided for a very expansive policy, but with very modest effects on monetary aggregates. This is partly due to the fact that excess reserves have increased drastically (von Hagen (2009)) as a result of the interbank market failure. Also the Fed's decision to pay interest on excess and required reserves increased the incentives for banks to hold more excess reserves.

Commercial banks will manage the portfolio volume by adjusting borrowed reserves L_c. As eq. (16) shows, a change in D leads to an adjustment of L_c which also determines the money base. The multiplier (24) is therefore valid only in case of a fixed L_c. Otherwise an expansion of the money base by $dS > 0$ induces a decrease of L_c because the bank attempts to keep its portfolio volume on an optimal level. In an equilibrium ("long run") perspective the values of D and L_c are determined by the solution of the linear equation system (16) and (23):

$$D^{\text{long}} = \begin{cases} (S - \overline{BC}) + (1-\lambda_E)\left(\frac{(1+\lambda_R\mu_R - \frac{1}{2}\theta\lambda_R^2\sigma_R^2)^{(1-\theta)}}{\rho_c}\right)^{1/\theta} & \text{for } L_c^{\text{long}} > 0 \\ \frac{S-\lambda_E\overline{BC}}{r+(1-r)\lambda_E} & \text{for } L_c^{\text{long}} = 0 \end{cases} \tag{25}$$

where L_c^{long} is given as

$$L_c^{\text{long}} = \arg\max \left\{ (r + (1-r)\lambda_E) \left(\frac{(1 + \lambda_R \mu_R - \frac{1}{2}\theta\lambda_R^2\sigma_R^2)^{(1-\theta)}}{\rho_c} \right)^{1/\theta}, \ 0 \right\} \tag{26}$$

When the commercial bank is able to keep the portfolio volume at the chosen optimal level, the required reserve rate r does not play a role anymore. The central bank is then able to enforce an increasing money supply D but it is not able to initiate a multiplier process since we have

$$\frac{dD^{\text{long}}}{dS} = 1$$

The conventional multiplier (24) holds true only if the banks are not willing or able to adjust L_c. If L_c is determined endogenously by optimal banking behavior, then outright purchases of securities seem to be an ineffective way for initiating a money multiplier process and should therefore be considered for finetuning operations only.

The second policy variable of the central bank are the refinancing conditions ρ_c. A change in ρ_c affects the demand for L_c and therefore the money base. We do not distinguish different types of borrowed reserves like standing facilities and open market operations on the market for reserves – all reserves where commercial banks have to pay interest rates to the central bank are subsumed to central bank loans L_c. It is easy to derive money multipliers for changes in the rate ρ_c in the same mechanistic way as before. In the short run L_c is assumed to respond only to changes in ρ_c. Ceteris paribus we have from (16) and (23):

$$\frac{dD}{d\rho_c} = \frac{dD}{dL_c}\frac{dL_c}{d\rho_c} = -\frac{1-\lambda_E}{\theta\rho_c(r + \lambda_E(1+r))} \cdot \left(\frac{(1 + \lambda_R \mu_R - \frac{1}{2}\theta\lambda_R^2\sigma_R^2)^{(1-\theta)}}{\rho_c} \right)^{1/\theta} < 0 \tag{27}$$

In the long run, ρ_c affects the equilibrium values D^{long} and L_c^{long} which yields the multiplier:

$$\frac{dD^{\text{long}}}{d\rho_c} = -\frac{1-\lambda_E}{\theta\rho_c} \cdot \left(\frac{(1 + \lambda_R \mu_R - \frac{1}{2}\theta\lambda_R^2\sigma_R^2)^{(1-\theta)}}{\rho_c} \right)^{1/\theta} < 0 \tag{28}$$

in case of $L_c^{\text{long}} > 0$ in the long run and, of course, $dD^{\text{long}}/d\rho_c = 0$ otherwise. Note, that also in case of a fixed L_c due to VaR restrictions the multiplier becomes zero. Comparing (27) and (28), it can be seen that in the long run the impact of monetary policy is lower than in the short run.

5.2 The impact of the non-banking sector

The previous analysis has shown that the impact of monetary policy on monetary aggregates is strongly influenced by bank's decision on portfolio structure and reserve demand. A severe shortcoming of a mechanistic multiplier analysis is that interest artes i, ρ are taken as given. However, they will change as a result of changed loans supply L and bonds demand B. Therefore the analysis is incomplete. Furthermore, multiplier analysis presumes that deposits are automaticcally created by loans rather than being an autonomous decision of non-banks. Since holdung deposits D and demanding loans L are driven by different micro-motives of households and firms, a mechanistic multiplier analysis is not very informative.

We will now consider the behavior of non-banks, following the same logic as in section 4. To obtain a consistent framework of balance sheets we assume that central bank's securities are bonds: $S = B^{cb}$. The upper index denotes the sector or institution which holds the bonds. Hence, the balance sheets of the central bank, the commercial bank, and their aggregated balance sheets reads:

central bank	$B^{cb} + L_c$	$=$	$E + rD$
commercial banks	$B^b + L + E$	$=$	$(1-r)D + L_c + \overline{BC}$
aggregated	$B^{cb} + B^b + L$	$=$	$D + \overline{BC}$

The non-bank sector consist of firms and households. We assume that firms hold physical capital PC and money D^f as assets, and loans L^f, bonds $\bar{B}$, and capital $\bar{C}$ as liabilities. Households hold bonds B^h, firm and bank capital, and money D^h. On the liability side we have the net financial wealth NFW and loans L^h. Thus we have:

firms	$PC + D^f$	$=$	$\bar{B} + \bar{C} + L^f$
households	$\bar{C} + \overline{BC} + B^h + D^h$	$=$	$NFW + L^h$
aggregated	$PC + D + \overline{BC}$	$=$	$NFW + (\bar{B} - B^h) + L$

with $D = D^h + D^f$ and $L = L^d = L^f + L^h$ (loans demand) and $\bar{B} = B^{cb} + B^b + B^h$. Therefore, adding all balance sheets of the bank and non-bank sector leads to the identity of physical capital goods and net financial wealth, implying that investments equals savings. Observe, that in case of subtracting $PC = NFW$ from the aggregated non-bank balance sheet, L is not neccessarily equal to D since banks and non-banks are connected also by equity and bonds contracts. Holding deposits D and demanding loans L are based on different motives. It is then not reasonable anymore to consider a "money market". The focus has to be on the credit and bonds market (see Palley (2008b)).

Without going into details of the micro-motives of households and firms, we consider

$$\begin{aligned} B^h &= B^h(\underset{-}{\rho}, \underset{+}{i}) \\ D &= D(\underset{-}{\rho}, \underset{-}{i}, \underset{+}{y}) \\ L^d &= L^d \underset{-}{\rho}, \underset{+}{i}, \underset{+}{y}) \end{aligned} \tag{29}$$

which is in line with standard economic literature.

Note, that all *plans* of banks, firms, and households about their asset and liability side could be consistently realized only in case of equilibrium interest rates. On the loans market we have in a partial equilibrium

$$L(\rho^*, i, \rho_c) = L^d(\rho^*, i, y)$$

and therefore $\rho^*(i, \rho_c, y)$. On the bonds market we have in partial equilibrium

$$\bar{B} = B^{cb} + B^b(\rho, i^*, \rho_c) + B^h(\rho, i^*)$$

and therefore $i^*(\rho, \rho_c, B^{cb})$. Due to this interdependency of the two markets, both interest rates are positively related and the total equilibrium values are $\rho^{**}(\rho_c, y, B^{cb}), i^{**}(\rho_c, y, B^{cb})$. The partial derivatives for ρ_c and y are positive, and negative for B^{cb}. Observe, that $i^{**}(\cdot, y)$ is a kind of LM curve which is parametrized by central bank policy variables.

Instead of relying on a mechanistic multiplier process, the central bank is only able to influence credit and deposit volume by affecting the interest rates. Contrary to some Post Keynesian views where money is endogenous but the interest rate is exogenously determined by the central bank, we take money *and* interest rates as endogenous. Interest rates determine the decisions about holding bonds or deposits, and the supply and demand of loans. If the real sector is taken into account, things become more complicated: Assume a negative relationship between income y and the interest rate i according to the IS curve. Then monetary policy has a *direct* influence on money demand $D(y, i)$ via the changed interest rates, but also an *induirect* influence via real effects on y. An appropriate analysis of monetary policy effects on monetary aggregates has to replace the mechanistic multiplier approach by

$$\frac{dD}{d\rho_c} = \frac{\partial D}{\partial i}\frac{\partial i^{**}}{\partial \rho_c} + \frac{\partial D}{\partial y}\frac{\partial y}{\partial i}\frac{\partial i^{**}}{\partial \rho_c} \tag{30}$$

and
$$\frac{dD}{dB^{cb}} = \frac{\partial D}{\partial i}\frac{\partial i^{**}}{\partial B^{cb}} + \frac{\partial D}{\partial y}\frac{\partial y}{\partial i}\frac{\partial i^{**}}{\partial B^{cb}} \tag{31}$$

where the first terms on the r.h.s. include all direct effects within the fiancial sector as discussed in this paper, while the second terms denote indirect effects from the real sector. It is a priori not clear which of both effects dominates the results.

The influence of the real sector on the monetary aggregate D runs as follows: Increasing income y leads to an increased loans demand. This affects the loans interest rate and via portfolio adaptations also the bonds interest rate which dampens money demand. A change in deposits as well as changed interest rates have an effect on the reserve demand. It is a matter of the monetary policy strategy how the central bank responds to such a shock (e.g. by accomodation). This will not be analysed in this paper since the purpose has been to demonstrate how the monetary aggregate is determined endogenously be the behavior of banks and non-banks, and how these processes are influenced by policy variables B^{cb} and ρ_c.

5.3 Numerical examples

From simple optimization considerations we obtain functions which depend in a non-linear and partially non-monotonous way on the variables. The explicit form of the behavioral equations for optimal loans supply L, bonds demand B^b, and demand for borrowed reserves L_c are too complicated to show them here. They consist of optimal structural variables $\tilde{\lambda}_L, \lambda_R$ (determining $\lambda_L, \lambda_B, \lambda_E$) and optimal reserve demand L_c which determines the volume.

Figure 3 shows the relevant structural variables which have slopes that could be expected intuitively. The underlying values are $p = 0.95, \sigma_B^2 = 0.2, r = 0.05, \theta = 1.5, D = 5, \overline{BC} = 1, \rho_c = 0.02$. They are truncated to the interval $[0, 1]$, neglecting any VaR constraints. Note that for very small interest rates ρ the expected return could be negative due to the possibility of total debt failure. Therefore $\tilde{\lambda}_L$ may have a zero border value. Such truncations also affect λ_L, λ_B and hence L_c, L, B, E, causing the kinks in the 3D-plots.

Figure 4 shows L_c and the resulting functions for L, B, E. Observe, that the bonds demand depends non-monotonously on ρ. For low loans interest rates the portfolio will consist only of bonds. When ρ increases, the portfolio will be restructured in favor of loans. But also the fraction λ_R will increase because the risky part of the portfolio becomes more attractive. Therefore the bonds demand increases. With high values of ρ we have $\lambda_R = 1$ so that the restructuring effect within the risky part of the portfolio dominates and the bonds demand will decrease again, causing a non-monotonicity. Another non-monotonous dependency is possible, though not observed with these parameter values: If i (ρ) increases, bonds (loans) become more attractive, but we may have countervailing effects because the marginal utility of the portfolio and hence the portfolio volume decreases. If the latter effect dominates, bonds demand (loans supply) could decrease although interest rates increase.

[Figures 3-4 about here]

6 Discussion

In this paper we presented a simple model of the aggregated commercial banking sector. The bank is assumed to manage its assets and liabilities according to risk, return, and liquidity considerations. This is done by the portfolio as well as by the Value at Risk methodology. Although each loan creates deposits by the initial accounting record, the non-bank sector decides due to different motives about demanding loans and holding deposits. The existence of other financial contracts than loans (here: bonds and equity) allows for distinct decisions about loans demand and holding deposits and a inequality of L and D.

From the banking side the endogeneity of money supply is hence twofold: The *structure* of the portfolio, driven by risk and liquidity preferences, determines the money multiplier, while the acchieved portfolio *volume* determines the demand for reserves and hence the money base. Nevertheless we made clear that multiplier analysis is of very limited use because it neglects the behavior of the non-bank sector. An analysis of money endogeneity has to abandon the multiplier view.

The commercial banking sector is a hinge between the central bank policy and the non-banking sector. As we have outlined in the section 4.2 the demand for money D and for loans L is determined by the dispositions of non-banks, not by a multiplier. This part of the endogenous money creation process will be studied in more detail in a subsequent paper. We haven't made any assumptions about central bank behavior. The central bank has to respond to shocks from the banking and non-bank sector in some way, e.g. by accomodating positive credit demand shocks. It is also possible to incorporate fixed policy rules like the Taylor rule into this framework. However, it has to be clarified that the central bank's decision variable is not *the* interest rate, but ρ_c which has an *influence* on ρ and i in a non-linear way. Such a monetary policy analysis deserves further investigation.

As an extension of the framework different types of liabilities may be considered. Deposits D have been assumed to have no interest rate and a high risk of outflows and (therefore) a requirement for holding excess reserves. However, there are other types of liabilities where the bank has to pay interest rates but has no reserve requirements. This enriches the strategic possibilities to attrract deposits in order to enhance the portfolio volume and is hence a substitute for the central bank loans demand L_c. Since these liabilities have no reserve requirement, this may have substantial effects on the money multiplier – the abilities of the central bank to manage the expansion process are much more restricted as they are anyway.

A further extension would be to endogenize the bonds supply since bonds are an imperfect substitute to loans. For a more dynamic perspective it would be desirable to include expectations explicitely into the calculus and to consider the different maturities of the liabilities and assets. Thus the banks are prevented from an instantanous adaption of their portfolio when the environment changes.

References

Bajec, L. and Graf Lambsdorff, J. (2006), 'There Is No Bank Lending Channel!', *Passau Economic Discussion Papers 46*.

Bernanke, B. and Blinder, A. S. (1988), 'Credit, Money, And Aggregate Demand', *American Economic Review, Papers And Proceedings 78*, pp. 435–439.

Bofinger, P. (2001), *Monetary Policy: Goals, Institutions, Strategies, And Instruments*, Oxford.

Chick, V. and Dow, S. (2002), 'Monetary Policy with ndogenous Money and Liquidity Preference: a Nondualistic Treatment', *Journal of Post Keynesian Economics 24*(4), pp. 587–607.

Fontana, G. (2004), 'Rethinking Endogenous Money: A Constructive Interpretation Of The Debate Between Horizontalists And Structuralists', *Metroeconomica 55*(4), pp. 367–385.

Hellmann, T. and Stiglitz, J. (2000), 'Credit And Equity Rationing in Markets With Adverse Selection', *European Economic Review 44*, pp. 281–304.

Huang, C.-F. and Litzenberger, R. H. (1988), *Foundations for Financial Economics*, North-Holland.

Lavoie, M. (1992), *Foundations of Post-Keynesian Economic Analysis*, Edward Elgar, Aldershot.

Lavoie, M. (2006), 'A Post-KeynesianAmendment to the New Consensus on Monetary Policy', *Metroeconomica 57*(2), pp. 165–192.

Mankiw, N. G. and Romer, D. (1991), *New Keynesian Economics. Vol. 1: Imperfect competition and sticky prices + Vol. 2: Coordination Failures and Real Rigidities*, MIT Press, Cambridge Mass./London.

Mishkin, F. S. (2006), *The Economics of Money, Banking, and Financial Markets*, Addison-Wesley, Boston et al.

Moore, B. J. (1988), *Horizontalists and verticalists: the macroeconomics of credit money*, Cambridge University Press, New York.

Palley, T. I. (2002), 'Endogenous Money: What It Is And Why It Matters', *Metroeconomica 53*, pp. 152–180.

Palley, T. I. (2008*a*), Endogenous Money: Implications for the Money Supply Process, Interest Rates, and Macroeconomics, Working Paper 178, Political Economy Research Institute Working Paper.

Palley, T. I. (2008*b*), Macroeconomics without the LM: A Post-Keynesian Perspective, Working Paper 179, Political Economy Research Institute Working Paper.

Pollin, R. (1991), 'Two theories of money supply endogeneity: some empirical evidence', *Journal of Post Keynesian Economics 13*(3), pp. 366–396.

Rochon, L.-P. (1999), *Credit, Money, and Production: An Alternative Post-Keynesian Approach*, Edward Elgar, Cheltenham.

Romer, P. (2000), 'Keynesian Macroeconomics Without The LM Curve', *Journal Of Economic Perspectives 14*, pp. 149–169.

Stiglitz, J. and Weiss, A. (1981), 'Credit Rationing in Markets With Impferfect Information', *American Economic Review 17*, pp. 393–410.

Taylor, J. B. (1993), 'Discretion versus Policy Rules in Practice', *Carnegie-Rochester Conference Series on Public Policy 39*, pp. 195–214.

Wahl, J. E. and Broll, U. (2003), 'Value at Risk and Bank Equity', *Jahrbücher für Nationalökonomie und Statistik 223*, pp. 129–135.

Woodford, M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.

Wray, L. R. (2007), Endogenous Money: Structuralist and Horizontalist, Working Paper 512, The Levy Economics Institute Working Paper Series.

von Hagen, J. (2009), The Monetary Mechanics of the Crisis, Working Paper 2009/08, Bruegel Policy Contribution.

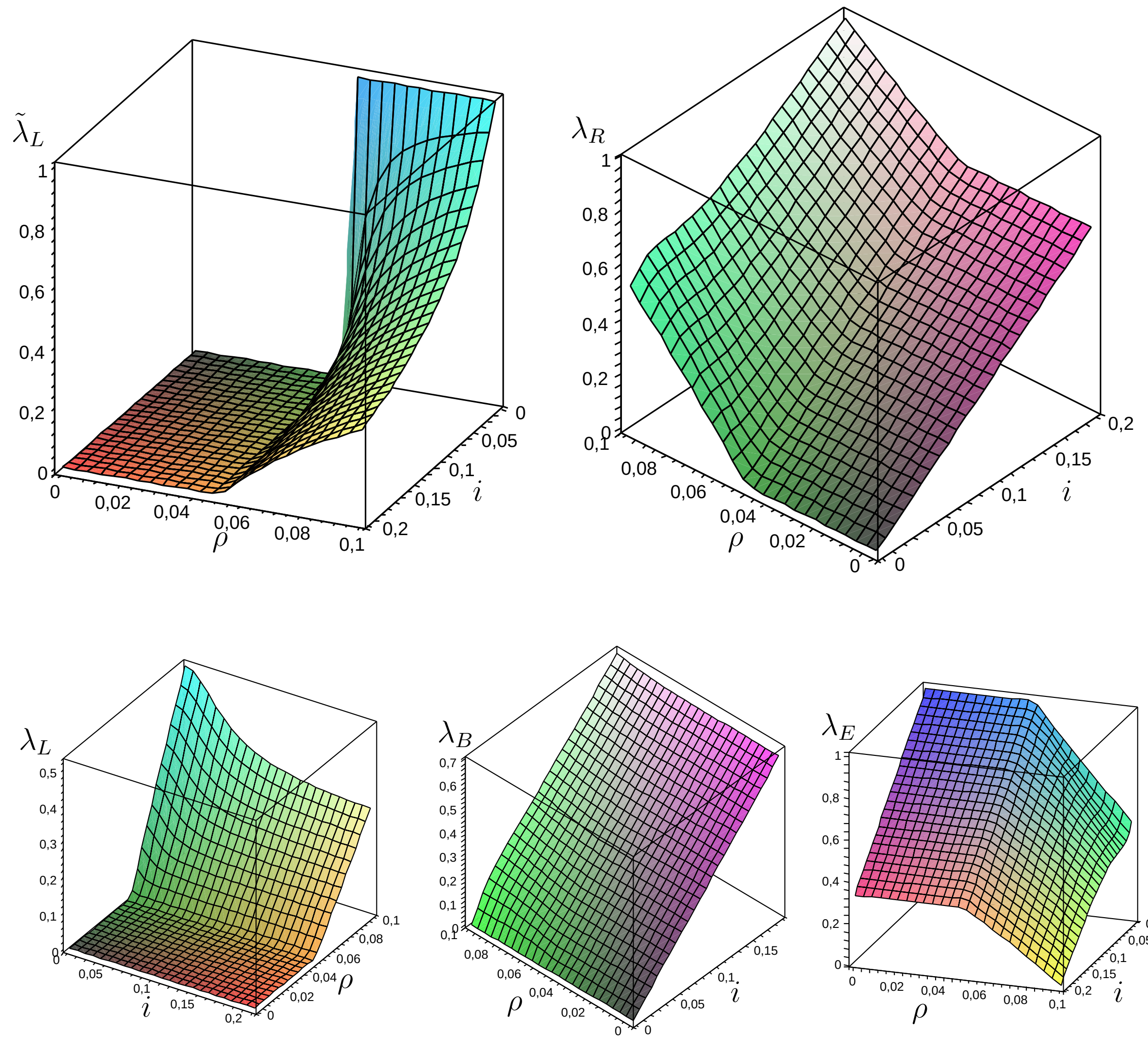

Figure 3: *Structural variables: structure of the risky portfolio* $\tilde{\lambda}_L$ *and share of the risky portfolio* λ_R *(first row), resulting shares* $\lambda_L, \lambda_B, \lambda_E$ *of the total portfolio (second row)*

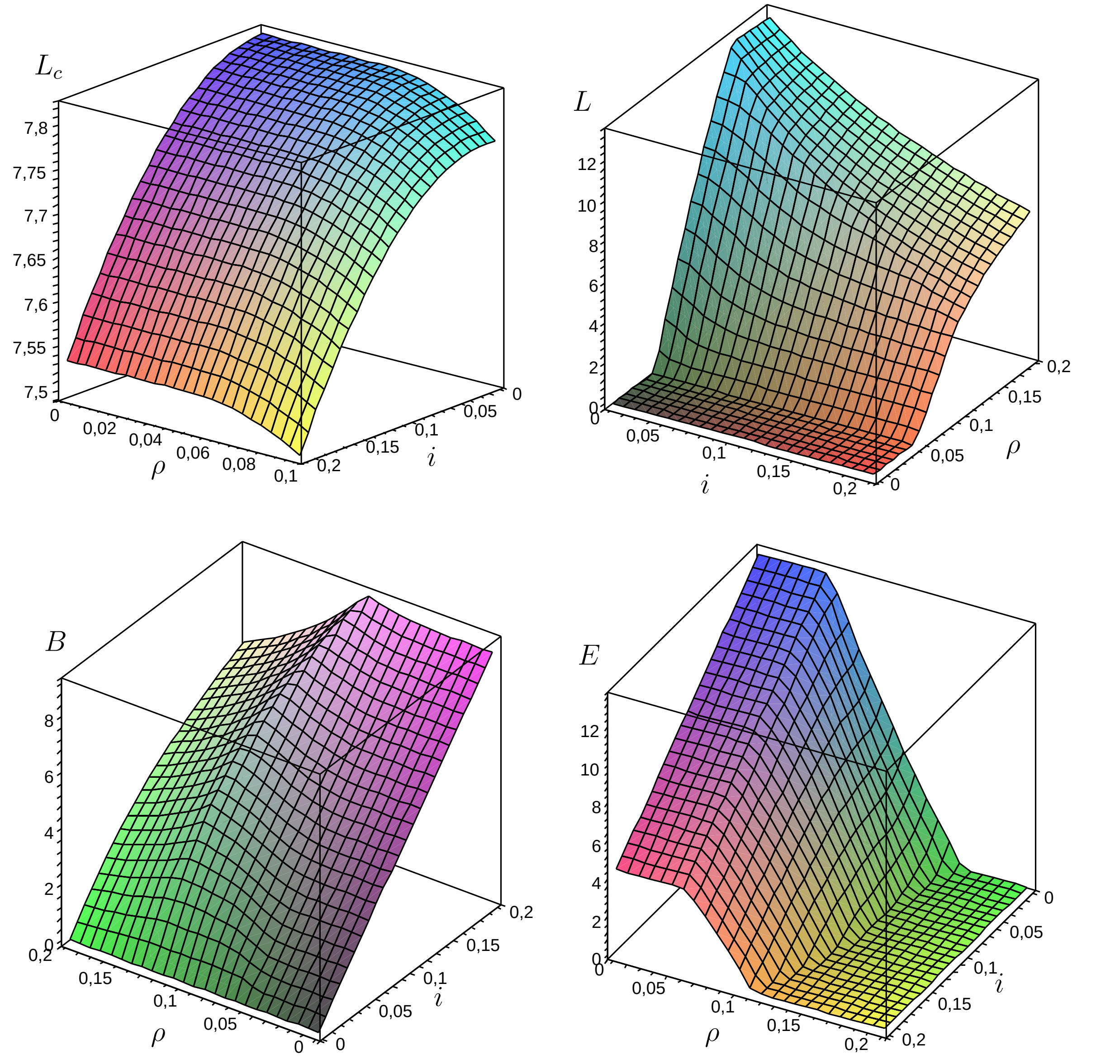

Figure 4: *Behavioral functions: Reserve demand L_c, credit supply L, bonds demand B, excess reserve demand E (with fixed deposits D)*

Appendix

The uncertain value of the portfolio is given by $\tilde{V} = V + \lambda V r$. The utility of the uncertain wealth is givven by a CRRA utility function: $u(\tilde{V}) = \tilde{V}^{(1-\theta)}/(1-\theta)$. It is not trivial to compute directly the expected utility $E[u(\tilde{V})]$, so we calculate the risk premium ψ and maximize the utility of the security equivalent:

$$\max_{\lambda_R \in [0,1]} u(V - \psi) = E[u(\tilde{V})]$$

Since $E[\lambda_R V r] = \lambda_R V \mu_R$ and $Var[\lambda_R V r] = \lambda_R^2 V^2 \sigma_R^2$ it is well known from literature that the risk premium is approximate

$$\begin{aligned}\psi &\approx -\frac{1}{2}\frac{u''(V)}{u'(V)}\lambda_R^2 V^2 \sigma_R^2 - \lambda_R V \mu_R \\ &= \frac{1}{2}\theta \lambda_R^2 V \sigma_R^2 - \lambda_R V \mu_R\end{aligned}$$

Inserting ψ into the utility function yields

$$u = \frac{(V(1 + \lambda_R \mu_R - \frac{1}{2}\theta \lambda_R^2 \sigma_R^2))^{(1-\theta)}}{1-\theta} \tag{32}$$

Maximizing this expression with respect to λ_R leads to the well known portfolio result:

$$\lambda_R = \min\left\{\frac{\mu_R}{\theta \sigma_R^2}, 1\right\}$$

which is independend from the invested portfolio volume V.