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by

**Werner Güth**

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Friedrich Schiller University Jena  
Carl-Zeiss-Str. 3  
D-07743 Jena  
[www.uni-jena.de](http://www.uni-jena.de)

Max Planck Institute of Economics  
Kahlaische Str. 10  
D-07745 Jena  
[www.econ.mpg.de](http://www.econ.mpg.de)

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# Prior-Free Optimality and Satisficing - A Common Framework and its Experimental Implementation -

Werner Güth\*

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## Abstract

Similar to welfare economics where with(out) interpersonal comparisons one defines unique (set-valued) welfare (Pareto) optima, we present a framework for one-person decision making where with(out) a prior probability distribution individual optimality prescribes usually a unique (set of) choice(s). Satisfiable aspirations in the sense that there exists some choice guaranteeing them define a much larger choice set whose intersection with the set of prior-free optimal choices is never empty. We also review experimental procedures and results which incentivize aspiration formation and reject even prior-free optimality experimentally.

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\*Max Planck Institute of Economics, Strategic Interaction Group, Kahlaische Str. 10, 07745 Jena, Germany, email: gueth@econ.mpg.de, phone: +49 3641 686 629, fax: +49 3641 686 623

## 1. Introduction

Imagine an agent who has to decide what a society with many agents should choose but who does not know his role in that society, i.e., who must choose behind the veil of ignorance. Such an agent may form a prior, specifying a probability for each role in society, and simply maximize his expected payoff based on this prior, e.g. the one assigning equal weights to all roles (Harsanyi, 1955) or the one giving all the weight to the worst position in society (Rawls, 1971). If the agent, however, does not rely on a prior, all what can be said about optimality is that he should not choose an option for which another option exists which makes everybody better off (or, in case of weak (Pareto-)efficiency, can improve one agent's lot without making others' worse). It is well-known that with(out) a prior one usually obtains unique (set-valued) welfare optima where the main purpose of a prior in this context is, of course, to allow for interpersonal utility comparisons.

The situation, discussed above, is just one special example of stochastic one-person decision making where the uncertainty is about which state of nature will be realized after decision making, e.g. the possible role in society, which the decision maker might assume, or just one of several possible states of nature in the more general context. Whether in welfare economics one accepts or rejects interpersonal utility comparisons, can thus be more generally conceptualized as whether in decision theory one wants to define optimal choices with or without assuming a prior probability distribution over the states of nature. The interpersonal trade-offs in welfare economics are thus a special case of "interstate" trade-offs, i.e., whether one wants to achieve more in one rather than in other states, in stochastic decision making. As a consequence, one either relies on expected utility maximization, what usually implies unambiguous optimal choice behavior, or on - what we call - prior-free optimality meaning that there exists no other choice with better results in all states of nature (or with a better result for one state and no worse ones for the other states in case of the weak notion).

When testing optimality empirically, a prior-free definition is of great advantage since it avoids all the problems of belief elicitation. It also renders optimality

more comparable to the satisficing approach for which the independence of belief and goal formation is more questionable. Assuming aspiration formation for each state of nature, we suggest a prior-free definition of satisfiability requiring that for at least one possible choice the aspirations for all states of nature can be satisfied. If satisficing is possible, the set of satisficing choices overlaps with that of the prior-free optimal ones. This allows to observe satisficing, meaning that the used choice option guarantees the aspiration for each and every state, without prior-free optimality as in the experiments of Berninghaus et al. (2006) and Güth et al. (2007). But, of course, satisficing may be impossible, e.g. when aspirations are too ambitious, whereas prior-free optimality is always possible.

One conceptual discussion hopefully contributes to the systematic attempt (see also Güth, 2007) to render the satisficing approach more applicable by eliciting aspirations<sup>1</sup>, similar to the revealed preference approach (Samuelson, 1938, and Varian, 1992), and by using them to define formally what satisfiability and satisficing means. To demonstrate applicability, we will review some experimental studies, relying on quite different experimental methods of eliciting state-specific aspirations, and suggest to incentivize the formation of aspirations. This does not only improve (the learning of) satisficing but also unambiguously rejects prior-free optimality, a more basic optimality concept than expected utility maximization.

In section 2, we more formally develop the unifying framework of prior-free optimality and satisfiability as well as satisficing. Section 3 discusses related experimental procedures and some findings. Section 4 concludes.

## **2. State-specific goal formation**

To simplify matters, actually to avoid any measure theoretic jargon, we will assume finitely many states of the world and also illustrate our approach by simple

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<sup>1</sup>The usual approach has been to infer aspirations from choice data, e.g. Seale and Rapoport, 1997 and 2000.

examples like a society with only two roles when choosing behind the veil of ignorance<sup>2</sup>. The essential ingredients of the formal setup are the set  $C$  of choices  $c$  with at least two elements, the (finite) set  $S$  of states  $S_i$ , and a function  $U_i(c)$  assigning to each choice  $c$  and state  $S_i$  a success or payoff level where the interpretation of  $U_i(c)$  will depend on whether we rely on the rational choice or the satisficing approach (see the related terminology of Savage, 1954, for an infinite state space).

For the rational choice interpretation, the choice set  $C$  is determined by the rules (of the decision problem or one-person game). In view of the satisficing approach, the decision maker may not actually perceive all possible choices as relevant alternatives but rather focus on a few obvious ones. In ultimatum experiments<sup>3</sup>, for instance, the proposer participants often seem to consider just the equal split, the smallest possible offer, and some intermediate offer which they apparently view as sufficient to render rejection a too costly choice for the responder.

Let  $S = \{S_1, \dots, S_n\}$  with  $n \geq 1$  denote the finite set of states  $S_i$ . When selecting a choice alternative  $c \in C$ , possibly a multi-dimensional choice vector, the decision maker does know the set  $S$  but not which state  $S_i \in S$  will occur. We do not endorse any prior governing the selection of a state  $S_i \in S$ , but assume that all  $n$  states are (expected<sup>4</sup> to be) possible and thus taken seriously when forming goals.

Since choices  $c \in C$  are made before the state  $S_i \in S$  is selected, when deciding, one has to anticipate what the choices imply for each and every state of nature. As general terminology, we speak of a payoff profile  $U = (U_i)_{1 \leq i \leq n}$  as a vector of payoffs  $U_i$ , one for each state  $S_i \in S$  and refer to  $U(c) = (U_i(c))_{1 \leq i \leq n}$  as the payoff profile, implied by the choice  $c \in C$ . Payoff is interpreted as (material) success

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<sup>2</sup>Assume, for instance, an exchange economy with two households and that the decision maker can determine the allocation and thus the trade amounts behind the veil of ignorance, i.e., without knowing which household he will be (for details see Hildenbrand and Kirman, 1976).

<sup>3</sup>The proposer participant can suggest how to share a positive amount of money what the responder participant can accept and thereby implement the payoff proposal or reject, meaning that both receive nothing.

<sup>4</sup>For an at most boundedly rational decision maker it may be too difficult to understand which states have objectively 0-probability or are even impossible.

like profit when the decision maker is a firm. When speaking of optimality it is assumed that utility is monotonically increasing in payoff.

For a given choice set  $C$ , we say that  $c^* \in C$  is prior-free optimal if there exists no alternative choice  $c \in C$  such that  $U(c) \geq U(c^*)$  and  $U(c) \neq U(c^*)$ , i.e., no component  $U_i(c)$  of  $U(c)$  is smaller and at least one is larger than the corresponding component  $U_i(c^*)$  of  $U(c^*)$  or - in the strict version - if no alternative choice  $c$  satisfies  $U_i(c) > U_i(c^*)$  for all states  $S_i \in S$ . As when arguing for (Pareto-)efficiency in welfare economics, the convincing argument is that one can get more in at least one state without having to give up something in other states where the analogy is between states and individual roles in society. In both cases suboptimality can be avoided without sacrifice.

Clearly, in deterministic choice tasks with  $n = 1$ , this definition coincides with utility maximization. If for  $n \geq 2$  and any generic prior on  $S$  the choice  $c^* \in C$  maximizes the expected utility over the given choice set  $C$ , it is also prior-free optimal. The reverse, however, does not hold, simply because the more basic concept of optimality does not require a prior which is so crucial when maximizing expected utility. As efficiency without interpersonal utility comparisons prior-free optimality will usually imply a large set of optimal choices. Only when selecting one of them by a process of goal formation for each state, the decision maker encounters the trade-off whether to demand more in one state at the cost of getting less in other states.

When referring to bounded rationality, we rely on payoff profiles  $A = (A_i)_{1 \leq i \leq n}$  whose components are aspirations rather than utilities, meaning that  $A_i$  is the payoff or success the decision maker wants to achieve in state  $i = 1, \dots, n$ , and that we rely on satisficing rather than optimizing. For an aspiration profile  $A$  we say that it is satisfiable if there exists some choice  $c \in C$  with  $U_i(c) \geq A_i$  for all states  $i = 1, \dots, n$ , i.e., the actual payoffs  $U_i(c)$  of some choice  $c$  guarantee the aspiration levels  $A_i$  in all states  $i$ . Satisfiability in this sense does not require that the decision maker is aware of all possible choices  $c \in C$  nor, when knowing  $C$ , would compare all alternatives  $c$  in  $C$ . All what is needed is that he could find some choice  $c \in C$  which meets the satisficing condition  $U_i(c) \geq A_i$  for  $i = 1, \dots, n$ .

A decision maker with a satisfiable aspiration profile  $A$  is said to be satisficing when using a choice  $c \in C$  for which  $U_i(c) \geq A_i$  holds for all states  $i = 1, \dots, n$ . If  $c^* \in C$  is an optimal choice in the prior-free sense, we say that the payoff profile  $U(c^*) = (U_i(c^*))_{1 \leq i \leq n}$  generated by it is an optimal aspiration profile. The set of satisfiable aspiration profiles obviously contains the set of optimal aspiration profiles as a proper subset. Prior-free optimality is thus a refinement of bounded rationality in the sense of satisficing.

Let us demonstrate the idea by a simple example to which we will also refer when reviewing experimental studies. Assume that the decision maker can invest a given positive amount  $e$  of money, e.g. an interest free credit, in a riskless bond with a repayment rate  $r(> 1)$  and/or in a risky asset with a return rate of  $l$  with  $0 \leq l < 1$  in the bad state of nature  $S_1$  and of  $h$  with  $h > r$  in the good state of nature  $S_2$  where  $n = 2$ . The choice set is thus  $C = [0, e]$  with  $c \in C$  denoting the amount invested in the risky asset (and  $e - c$  in the riskless bond).

For an aspiration profile  $A = (A_1, A_2)$  satisfiability is granted if

$$(e - c)(r - 1) + c(l - 1) \geq A_1 \text{ and } (e - c)(r - 1) + c(h - 1) \geq A_2$$

or

$$\frac{er - A_1}{r - l} \geq c \geq \frac{A_2 - er}{h - r}.$$

Thus, if the lower bound for  $c$  exceeds its upper bound, the aspiration profile is too demanding and would have to be adapted. On the other hand, the interval for  $c$  can be very large if the two aspirations are too moderate (assume, for instance,  $A_1 = el$  and  $A_2 = er$ ). In this case, one could adapt by forming more ambitious aspirations. In our terminology only aspiration profiles  $A = (A_1, A_2)$  with

$$e \geq \frac{er - A_1}{r - l} = \frac{A_2 - er}{h - r} \geq 0$$

are optimal.

It is one thing to define what is meant by (un)bounded decision rationality but quite another task to derive behavior which is (un)boundedly rational. We have so

far abstracted from computational or procedural aspects of optimal or boundedly rational choice making (see also Simon, 1955, Selten, 1998, Güth, 2000, Berninghaus, Güth and Kliemt, 2006). Rather than to the familiar maximization of expected utility, derivation of prior-free optimality is computationally similar to deriving (Pareto-) efficient allocations: assuming feasible utilities for  $n - 1$  states  $S_i \in S$ , one maximizes the utility for the remaining state where, due to the monotonicity of utility in payoff, one can substitute utility by payoff.

To check satisfiability one has to determine the intersection of the  $n$  half-spaces<sup>5</sup>, defined by the  $n$  conditions that payoff guarantees the aspiration in a given state, what can be computationally very demanding. The idea of satisficing, however, does not assume that one checks satisfiability when forming aspirations. Rather one will form a preliminary aspiration profile, then search successively for satisficing choice alternatives, and adapt aspirations (see Sauermann and Selten, 1962) in the sense of more (less) ambitious goals when (not) quickly finding a satisficing choice.

### 3. Experimental procedures and results

In the following, we will briefly discuss experimental procedures and some findings that (allow to) reject prior-free optimality and, depending on the procedure used, partly question<sup>6</sup> and partly confirm the satisficing hypothesis. As for the rational choice approach, hardly anything can be concluded without specifying the goals that have been formed. We therefore do not report attempts trying to infer aspirations from choice data (e.g. Seale and Rapoport, 1997 and 2000).

Earlier and convincing experimental studies, directly asking for success aspirations in negotiations, e.g. Tietz (1996), and Tietz and Weber (1978) imposed a ladder

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<sup>5</sup>In the case of the investment task above, which can obviously be generalized (see Güth, 2007), the half-spaces are linear.

<sup>6</sup>In our view, it is better to specify the satisficing hypothesis more rigorously and run the risk of contradictory evidence than to view and use it as a flexible and intuitive "language" for ex-post explanations of decision behavior.



structure for each specific goal by questions like "what is your initial demand?", "when do you threaten to break off?", "when will you actually break off?", etc. After filling out the aspiration questionnaire, participants could quite freely negotiate so that one can compare bargaining behavior and success to aspirations. The robust finding was balancing of aspiration concessions, i.e., parties conceded in aspiration grid (yielding to the next lower aspiration level) equally often so that mostly the interacting parties achieved both either their second or third highest aspiration level.

In a series of experiments (see Güth, 2007, for a review), based on financial investment tasks similar to the one considered above, the arbitrary ladder structure for aspirations is avoided since aspirations are state specific. Although the states of nature could be naturally ordered from worst to best, participants were free to form the same aspiration levels for different states<sup>7</sup>. More important for our discussion is, however, that aspiration profiles  $A = (A_i)_{1 \leq i \leq n}$  were just cheap talk, i.e., before deciding participants were asked for state-specific aspirations without any financial consequences of their aspirations. Not surprisingly, the findings are ambiguous with usually not more than half of the participants being satisficing and nearly never behaving optimally in the sense of prior-free optimality and optimal aspiration profiles.

The obvious alternative is therefore to incentivize aspiration formation. Berninghaus et al. (2006) explored a setting with strategic interaction which maintained that states of nature can be ordered from worst to best. Participants earned their aspiration  $A_i$  for the actually realized state of nature  $S_i$  if their payoff  $U_i(c)$ , implied by their choice  $c$ , guaranteed it, i.e., if  $U_i(c) \geq A_i$ , otherwise they only earned the highest lower aspiration  $A_j$  with  $U_i(c) \geq A_j$ , formed for a worse state of nature  $S_j$ . Possibly due to the fact that strategic interaction renders the decision task much more difficult, the findings did not overwhelmingly confirm the satisficing hypothesis and not at all prior-free optimality and formation of optimal aspiration profiles.

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<sup>7</sup>The intuitive hypothesis that one will refrain from forming fully elaborated aspiration profiles when the number  $n$  of states is very large could not yet be convincingly tested since so far  $n$  was with  $n \leq 3$  too small.

In their experimental study of intertemporal allocation, Güth et al. (2007) avoided not only the complexity of strategic interaction but also the natural ordering of states. Participants could either have a short or a long "life" and enjoy either a low or a high future income what defines four different states of nature. In the incentivized treatment participants earned their aspiration for the realized state if their payoff for this state, implied by their choice  $c$ , guaranteed it; if not they earned nothing (except for their show up fee). A control treatment also asked for aspirations but did not incentivize their formation. Here the results were much more comforting for the satisficing hypotheses: after some learning, actually after the first round, nearly all participants of the "incentive treatment" were satisficers whereas participants of the control treatment with cheap-talk aspirations were systematically less satisficing. We conclude from this that one should incentivize the formation of aspirations as other choice behavior.

One may object that incentivizing aspiration formation, as described above, changes "the decision problem" since, in case of  $U_i(c) > A_i$  for the realized state  $S_i$ , one earns only  $A_i$  rather than  $U_i(c)$  and in case of  $U_i(c) < A_i$  much less than  $U_i(c)$ . But, of course, this does not question prior-free optimality and optimality of aspiration profiles and thus does not save the rational choice hypothesis from being persistently rejected by the studies reported. The main counter-argument, however, seems to be that this form of incentivized aspiration formation captures the usual interpretation of (payoff) aspirations, namely that one mainly cares to achieve what one has aspired, i.e., is quite frustrated when failing to do so, and is less interested in payoff increases between aspiration levels.

## 4. Discussion

The main advantage of prior-free (un)bounded rationality is its non-Bayesian aspect. Although we deal with uncertainty when  $n \geq 2$ , no prior probabilities for the states in  $S$  are required. Another advantage is that prior-free (un)bounded rationality can be tested experimentally without eliciting beliefs. If the decision maker uses some choice  $c \in C$  which is not prior-free optimal, i.e., there exist

$\hat{c} \in C$  with  $U(\hat{c}) \geq U(c)$  and  $U(\hat{c}) \neq U(c)$  (at least one component  $U_i(\hat{c})$  of  $U(\hat{c})$  is larger and no component of  $U(\hat{c})$  is smaller than the corresponding component  $U_i(c)$  of  $U(c)$ ), this contradicts optimality.

To rely on aspiration data  $A = (A_i)_{1 \leq i \leq n}$  as well as on choice data may be considered as strange. Earlier empirical, usually experimental studies of satisficing behavior mostly estimated aspiration levels from choice data rather than eliciting them directly. Although direct elicitation of aspirations is rare this does not mean that it is impossible. Our proposal is to incentivize aspiration formation in an experiment by asking participants for

- an aspiration profile  $A = (A_i)_{1 \leq i \leq n}$ ,
- a choice  $c \in C$ , and
- paying them only  $A_i$  for the realized state  $S_i$  if  $U_i(c) \geq A_i$  and considerably less if  $U_i(c) < A_i$ .

One can test the rationality hypothesis with the help of aspiration or choice data whereas satisfiability just requires aspiration data. Only when testing the satisficing hypothesis, one needs both, aspiration and choice data. Altogether the idea of state-specific and thus prior-free goal formation not only suggests a more basic notion of (un)bounded decision rationality but also new ways of testing such concepts what we have demonstrated by reporting some closely related experiments.

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