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Less fighting than expected — experiments with wars of attrition and all-pay auctions

Hannah Hörisch* and Oliver Kirchkamp†

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Abstract

We use experiments to compare dynamic and static wars of attrition (i.e. second-price all-pay auctions) and first-price all-pay auctions. Many other studies find overbidding in first-price all-pay auctions. We can replicate this property. In wars of attrition, however, we find systematic underbidding.

We study bids and revenue in different experimental frames and matching procedures and draw a link to the literature on stepwise linear bidding functions.

Keywords: War of attrition, dynamic bidding, all-pay auction, stabilisation, volunteer's dilemma, experiment

JEL classification C72, C92, D44, E62, H30

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1 Introduction

A war of attrition can be interpreted as a fight between two players about a prize. For each second during which players are fighting both players incur a cost. As soon as one player gives up, the other earns a prize. This situation is equivalent to a second-price all-pay auction. Both bidders make a bid (the amount of time that they are willing to hold out during the fight), the bidder with the highest bid wins the prize, and both bidders pay the amount of the second highest bid—the winner does not incur a cost once the loser gave up.

Wars of attrition have been used to model a large range of problems: Arms races (Zimin and Ivanilov, 1968), fights between animals (Bishop, Canning, and Smith, 1978; Riley, 1980), the voluntary provision of public goods (Bliss and Nalebuff, 1984; Bilodeau and Slivinski, 1996), competition between firms (Ghemawat and Nalebuff, 1985; Fudenberg and Tirole, 1986; Roth, 1996), the settlement of strikes (Kennan and Wilson, 1989; Card and Olson, 1995), fiscal and political stabilisations (Alesina and Drazen, 1991; White, 1995), the timing of exploratory oil drilling (Hendricks and Porter, 1996), and many others.

In this paper we want to use experiments to better understand wars of attrition. Though we are not aware of any experiment with wars of attrition there are two situations which have been studied experimentally: Rent-seeking contests and first-price all-pay auctions.

In a rent-seeking contest all bidders pay their own bid. The probability of winning the item is an increasing function of the bid. This situation has been introduced by Bishop, Canning, and Smith (1978) and Tullock (1980).[1] Possibly the first test of Tullock's model in the laboratory is an experiment by Millner and Pratt (1989). The, perhaps disappointing, result is that participants make even larger socially wasteful investments in the rent-seeking contest than they should in equilibrium. Several other experiments with rent-seeking contests followed[2]. We will give a brief overview in section 3.

Also in a first-price all-pay auction all bidders pay their own bid. In contrast to the rent-seeking contest the highest bidder obtains the prize with certainty.[3] Experiments with first-price all-pay auction have been done by Davis and Reilly

[1] In the formulation of Tullock first-price all-pay auctions actually are a special case.

[2] Shogren and Baik (1991), Davis and Reilly (1998), Potters, de Vries, and van Winden (1998), Vogt, Weimann, and Yang (2002), Anderson and Stafford (2003), Schmidt, Shupp, Swope, and Cardigan (2004), Schmidt, Shupp, and Walker (2004), Abbink, Brandts, Herrmann, and Orzen (2007)

[3] In Krishna and Morgan (1997) only this case is called an *all-pay auction.*

(1998), Potters, de Vries, and van Winden (1998), Barut, Kovenock, and Noussair (2002), Gneezy and Smorodinsky (2006), and Müller and Schotter (2007). A famous variant of a first-price all-pay auction is the dollar auction game (Shubik, 1971).

A generalised war of attrition, or a volunteer's dilemma, is an nth price all-pay auction with $n-1>1$ prizes.[4] A simple version of such a game has been studied experimentally by Diekmann (1993). Participants decide whether to provide a public good. Each player can only choose between two different bids, a low bid (waiting for somebody else to provide the public good) or a high bid (provision of the public good).

Bilodeau, Childs, and Mestelman (2004) study experiments of a sequential version of this game where players are fully informed about each other's bidding costs.

A common finding in most of these experiments is that bids are higher than in the risk neutral Bayesian Nash equilibrium.[5] Furthermore, Müller and Schotter (2007) find what they call a 'bifurcation of effort', a step-shaped bidding function where bidders with a high cost make small or no bids at all while bidders with a small cost bid too much.

In this paper we want to find out whether these properties carry over to wars of attrition. As a benchmark we replicate the standard first-price all-pay auction experiment in a situation with a single prize and two bidders. As expected, we find overbidding. Similar to Müller and Schotter (2007) we also find what they call bifurcated bidding functions. Then we look at a war of attrition which we implement both as a dynamic and as a static bidding process. We find underbidding in particular for the dynamic implementation of the war of attrition. Furthermore, we do not find bifurcated bidding functions in the war of attrition.

We will proceed as follows: In section 2 we derive the equilibrium bidding functions and summarise some results from the literature on first-price all-pay auctions. In section 3 we discuss the experimental procedure and present our setup. Sections 4 presents the results and section 5 concludes.

[4]See Bulow and Klemperer (1999) for a derivation of the equilibrium of such a game.

[5]Table 1 on page 8 provides an overview.

2 Hypotheses

2.1 Equilibrium in the war of attrition

In this section we derive the risk neutral Bayesian Nash equilibrium for the war of attrition with two bidders, i and j. We will call P the prize from winning the war of attrition. In the auction bidders make bids b by staying for up to b seconds in the auction unless the opponent leaves the auction earlier. If bidder i is staying for b seconds in the auction this bidder incurs a cost $b \cdot c_i$, where c_i is private information of bidder i and not known to bidder j. The other bidder only knows that c_i is drawn from a uniform distribution over $[\underline{c}, \bar{c}]$. We follow a standard approach and assume that the opponent of bidder i, bidder j with cost per second c_j, uses a decreasing bidding function $\beta^W(c)$ with inverse $\beta^{W(-1)}(\cdot)$. Bidder i does not know c_j, but can calculate the expected utility EU_i, given that i's bidding cost per second are c_i and bidder i bids up to b seconds. If bidders maximise a utility function $u(x)$ their expected utility is

$$\mathrm{EU}_i = \underbrace{\int_{\underline{c}}^{\beta^{W(-1)}(b)} u(-b\,c_i)\, f(c_j)\; dc_j}_{\substack{\text{bidder } i \text{ does not win} \\ \text{and pays the own bid } b}} + \underbrace{\int_{\beta^{W(-1)}(b)}^{\bar{c}} u\left(P - \beta^W(c_j)\, c_i\right) f(c_j)\; dc_j}_{\substack{\text{bidder } i \text{ wins } P \text{ and pays} \\ \text{the second highest bid } \beta^W}}\,. \tag{1}$$

We will start with the risk neutral case. Then we can replace $u(x)$ by x and take the first derivative $\partial\mathrm{EU}_i/\partial b$ and substitute $b = \beta^W(c_i)$ to obtain the first order condition

$$-\, c_i \int_{\underline{c}}^{c_i} f(c_j) dc_j - \frac{P \cdot f(c_i)}{\beta^{W\prime}(c_i)} = 0\,. \tag{2}$$

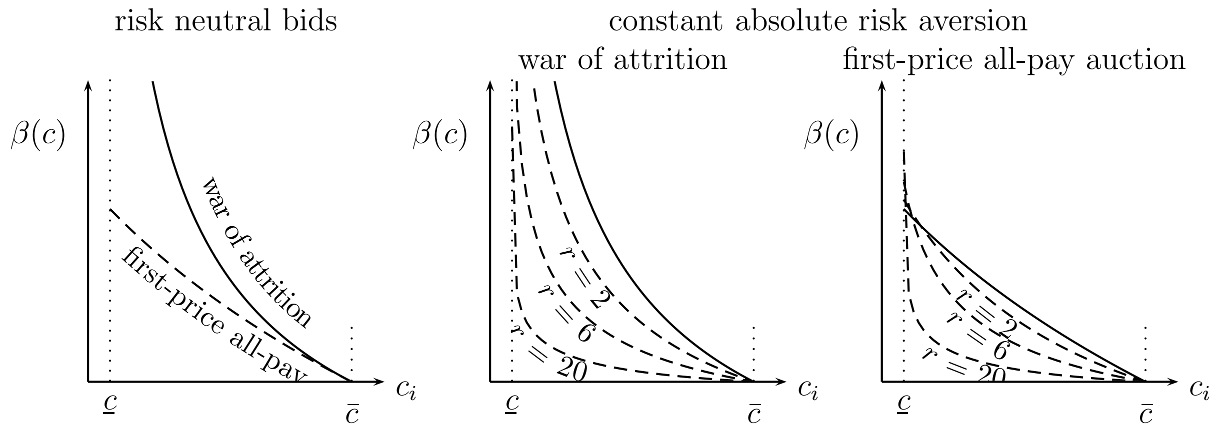
If c_i is uniformly distributed over $[\underline{c}, \bar{c}]$ we know that $f(c_i) = 1/(\bar{c} - \underline{c})$ and, hence, (2) can be simplified:

$$\beta^{W\prime}(c_i) = -\frac{P}{c_i(c_i - \underline{c})} \tag{3}$$

which, with the restriction $\beta^W(\bar{c}) = 0$, yields the equilibrium bidding function:

$$\beta^W(c_i) = \frac{P}{\underline{c}} \ln \frac{(\bar{c} - \underline{c})c_i}{(c_i - \underline{c})\bar{c}} \tag{4}$$

An example of the equilibrium bidding function is shown as the solid line in the



The left diagram shows risk neutral equilibrium bidding functions for the war of attrition (solid line) and first-price all-pay auction (dashed lines). The diagram in the middle shows equilibrium bidding functions for the war of attrition for bidders with constant absolute risk aversion and risk aversion parameters $r = 2$, $r = 6$, and $r = 20$. The diagram on the right shows equilibrium bidding functions for the first-price all-pay auction for bidders with constant absolute risk aversion and risk aversion parameters $r = 2$, $r = 6$, and $r = 20$.

FIGURE 1: Equilibrium bidding functions

left diagram in figure 1. The expected revenue is

$$R^W = \int_{\underline{c}}^{\bar{c}} \beta^W(c_i)\, 2\frac{c_i - \underline{c}}{(\bar{c} - \underline{c})^2}\, dc_i = P\frac{\bar{c} - \underline{c}\left(1 + \ln\frac{\bar{c}}{\underline{c}}\right)}{(\bar{c} - \underline{c})^2} \tag{5}$$

2.2 Equilibrium in the first-price all-pay auction

Similar to the derivation for the war of attrition in equations 1 to 4 we can find the equilibrium bidding function for the first-price all-pay auction. We assume that bidder j with cost c_j uses a decreasing bidding function $\beta^A(c)$ with inverse $\beta^{A^{(-1)}}(\cdot)$. The expected utility of bidder i with per period cost c_i who bids up to b periods is

$$\mathrm{EU}_i = \underbrace{\int_{\underline{c}}^{\beta^{A^{(-1)}}(b)} u(-b\, c_i)\, f(c_j)\; dc_j}_{\substack{\text{bidder } i \text{ does not win} \\ \text{and pays the own bid } b}} + \underbrace{\int_{\beta^{A^{(-1)}}(b)}^{\bar{c}} u\left(P - b\, c_i\right) f(c_j)\; dc_j}_{\substack{\text{bidder } i \text{ wins } P \text{ and pays} \\ \text{the own bid } b}}\; . \tag{6}$$

Again, we will first consider the risk neutral case $u(x) = x$. We take the first derivative $\partial\mathrm{EU}_i/\partial b$ and substitute $b = \beta^A(c_i)$ to obtain the first order condition

$$- c_i \int_{\underline{c}}^{c_i} f(c_j)dc_j - c_i \int_{c_i}^{\bar{c}} f(c_j)dc_j - \frac{f(c_i)}{\beta^{A'}(c_i)}P = 0 \tag{7}$$

With c_i being distributed uniformly over $[\underline{c}, \bar{c}]$ we have $f(c_i) = 1/(\bar{c} - \underline{c})$ and (7) can be simplified to

$$\beta^{A\prime}(c_i) = -\frac{P}{c_i \cdot (\bar{c} - \underline{c})} \,. \tag{8}$$

which, with the restriction $\beta^A(\bar{c}) = 0$ yields the equilibrium bidding function:

$$\beta^A(c_i) = \frac{P}{\bar{c} - \underline{c}} \ln \frac{\bar{c}}{c_i} \tag{9}$$

An example of the equilibrium bidding function is shown as a dashed line in the left part of figure 1. As long as $c_i \in (\underline{c}, \bar{c})$ we always have $\beta^A(c_i) < \beta^W(c_i)$. The expected revenue of the first-price all-pay auction is

$$R^A = \int_{\underline{c}}^{\bar{c}} \frac{\beta^A(c_i)}{\bar{c} - \underline{c}} \, dc_i = P \frac{\bar{c} - \underline{c}\left(1 + \ln \frac{\bar{c}}{\underline{c}}\right)}{(\bar{c} - \underline{c})^2} \tag{10}$$

Comparing with equation (5) we see that the expected revenue in the two auction formats is the same, $R^W = R^A$. Krishna and Morgan (1997) point out that this need not be the case if bidding cost are not distributed independently. They show that generally the revenue in the war of attrition is larger or equal than in the first-price all-pay auction.

2.3 Risk aversion

While we can not find a closed form solution for general utility functions, it is possible to follow the above steps also for specific utility functions. Constant relative risk aversion $u(x) = x^{1-r}$ is not very meaningful here since equilibrium payoffs can be positive and negative, thus, utility is sometimes real and sometimes complex, which is difficult to interpret. Here we consider only the case of constant absolute risk aversion $u(x) = -e^{-r \cdot x}$.

War of attrition With constant absolute risk aversion and a risk aversion parameter r the equilibrium bidding function becomes

$$\beta^{WR}(c_i) = \frac{\bar{c} - \underline{c}}{r\underline{c}} \left(1 - e^{\frac{rP}{\underline{c} - \bar{c}}}\right) \cdot \ln \frac{(\bar{c} - \underline{c})c_i}{(c_i - \underline{c})\bar{c}} \tag{11}$$

and the expected revenue is

$$R^{WR} = \frac{1 - e^{\frac{rP}{\underline{c}-\bar{c}}}}{r \cdot (\bar{c} - \underline{c})} \cdot \left(\bar{c} - \underline{c} \left(1 + \ln \frac{\bar{c}}{\underline{c}} \right) \right) . \tag{12}$$

In the risk neutral limit $r = 0$ these expressions coincide with their risk neutral counterparts given by (4) and (5). For all positive values of r bids and revenues are decreasing in r and smaller than the risk neutral values.

First-price all-pay auction Here the equilibrium bidding function becomes

$$\beta^{AR} = \frac{1 - e^{\frac{rP}{\underline{c}-\bar{c}}}}{r} \frac{\bar{c} - \underline{c}}{\underline{c} - \bar{c} \cdot e^{\frac{rP}{\underline{c}-\bar{c}}}} \cdot \ln \frac{\left((\bar{c} - c_i) \cdot e^{\frac{rP}{\underline{c}-\bar{c}}} + c_i - \underline{c} \right) \cdot \bar{c}}{(\bar{c} - \underline{c}) \cdot c_i} \tag{13}$$

and the expected revenue is

$$R^{AR} = \frac{\left(\underline{c} \cdot \ln \frac{\bar{c}}{\underline{c}} + rP \right) e^{\frac{rP}{\underline{c}-\bar{c}}} - \underline{c} \cdot \ln \frac{\bar{c}}{\underline{c}}}{r \cdot (e^{\frac{rP}{\underline{c}-\bar{c}}} \bar{c} - \underline{c})} \tag{14}$$

In the risk neutral limit $r = 0$ these expressions coincide with their risk neutral counterparts given by (9) and (10). For positive values of r bids are smaller if c_i is sufficiently large. This is consistent with the observation of Fibich, Gavious, and Sela (2006). In their paper risk is modelled in a different way, as 'weak risk aversion', which is a small perturbation of the risk neutral utility function. They also find that with increasing risk their 'low type', in our model the bidder with a high c_i, bids less the more risk averse bidders become. However, the 'high type', in our model the bidder with a low c_i, bids more the larger the amount of risk aversion. The intuition that is given by Fibich, Gavious, and Sela also applies in our case: Bidders with a high c_i will, most likely, not win the auction and just lose their bid. Thus, making a high bid is risky for them. The more risk averse bidders become, the lower the equilibrium bid. Bidders with a small c_i have good chances to win. Making a slightly too small bid can be very risky for them, thus, the more risk averse bidders become, the higher is the equilibrium bids for bidders with a low c_i. The right diagram in figure 1 shows bidding functions for different values of risk aversion r. We see that for most values of c_i risk averse bids are below the risk neutral bid. However, if c_i is very small, risk averse bids are higher than the risk neutral bid. The revenue is decreasing in r and smaller than the risk neutral. For the parameters that we are using R^{AR} decreases more slowly than R^{WR}, i.e. with risk averse bidders the expected revenue is larger with first-price all-pay auctions.

2.4 Overbidding in other experiments with all-pay auctions

Table 1 on page 8 provides an overview of experiments with first-price all-pay auctions and rent-seeking contests. Most of these experiments find systematic overbidding, i.e. bids that are higher than the risk neutral equilibrium. The table lists ten studies of rent-seeking contests. Seven of them find a significant amount of overbidding, two studies (Shogren and Baik, 1991; Vogt, Weimann, and Yang, 2002) find no significant deviation from equilibrium, and only one (Schmidt, Shupp, and Walker, 2004) finds significant underbidding in rent-seeking contests. Table 1 also lists five studies of first-price all-pay auctions. Four of these studies find a significant amount of overbidding while one (Potters, de Vries, and van Winden, 1998) finds no significant deviation from equilibrium. If we take first-price all-pay auctions and rent-seeking contests as a point of reference for our experiment on wars of attrition, then we should rather expect some overbidding in our experiment, too.

2.5 Bifurcated bidding functions

Müller and Schotter (2007) observe what they call bifurcations of bidding functions in experiments with first-price all-pay auctions. Compared with equilibrium bids, bids are too small if bidding is expensive, and too large if bidding is cheap. For each bidder Müller and Schotter identify a switching point from underbidding to overbidding. Bidding functions can be approximated with the help of a stepwise linear function similar to the one in figure 2. We will try to replicate their results and check whether and when these bifurcated bidding functions can also be found in a war of attrition in section 4.7.

3 Experimental setups

We will discuss our setup with the help of table 1.

Institution: As we can see from table 1 most experiments have been done with first-price all-pay auctions and rent-seeking contests. There is also one experiment with a volunteer's dilemma.

	institution	bidding procedure	heterogeneity among partici-pants over	number of contestants	number of prizes	repetitions in a group	periods	result
Millner and Pratt (1989)	rent-seeking	static	—	2	1	1	20	overbidding
Shogren and Baik (1991)	rent-seeking, framed in expected payoffs	static	—	2	1	32	32	close to equilib-rium
Davis and Reilly (1998)	rent-seeking / first-price	static	—	5	1	1	15	overbidding
Potters, de Vries, and van Winden (1998)	rent-seeking / first-price	static	—	2	1	1	30	overbidding in the rent-seeking case
Vogt, Weimann, and Yang (2002)	rent-seeking	sequential	—	2	1	1	1	efficient equilib-rium selected
Barut, Kovenock, and Noussair (2002)	first-price	static	valuation	6	2 or 4	1	20 or 50	overbidding
Anderson and Stafford (2003)	rent-seeking with entry-fee	static	cost, no. of contestants	1...10	1	1	1	overbidding
Schmidt, Shupp, Swope, and Cardigan (2004)	rent-seeking	static	—	2	1	1	5	overbidding
Schmidt, Shupp, and Walker (2004)	rent-seeking	static	—	4	1,3,∞	1	1	underbidding
Bilodeau, Childs, and Mestelman (2004)	volunteer's dilemma	dynamic	—	3	1	1	24–32	overbidding
Öncüler and Croson (2005)	risky rent-seeking	static	—	2,4	1	1	1	overbidding
Gneezy and Smorodinsky (2006)	first-price	static	—	4,8,12	1	10	10	overbidding
Abbink, Brandts, Herrmann, and Orzen (2007)	rent-seeking	static	—	2	1	20	20	overbidding
Müller and Schotter (2007)	first-price	static	cost	4	1,2	1,50	50	overbidding, bifurcation
our experiment	second-price / first-price	dynamic / static	cost	2	1	1,6	24	underbidding in the sequen-tial format, no bifurcation

Table 1: Comparison with other experiments

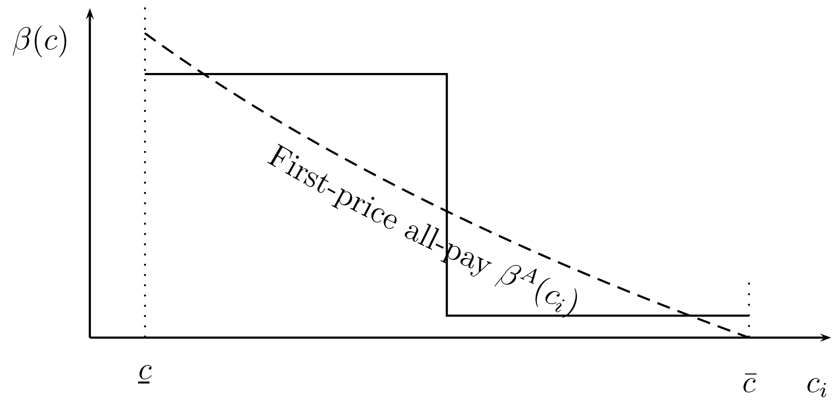


Figure 2: A bifurcated bidding function (solid) and an equilibrium bidding function (dashed)

In this paper we concentrate on wars of attrition. We use first-price all-pay auctions as a benchmark. At first sight the war of attrition seems to be very similar to a first-price all-pay auction. With our experiment we want to find out whether and how the differences between the two auction formats affects bidding behaviour and bidding anomalies.

The major source of uncertainty in a Tullock rent-seeking contest is the unknown choice of the opponent and the risk of the subsequent lottery. Rent-seeking contest may, thus, be regarded as a model of a contest where participants are ex ante rather symmetric and the determinants of success in the Tullock lottery can be interpreted as parameters which are revealed only ex post. First-price all-pay auctions and wars of attrition model a situation where a priory differences of talents or opportunities play the dominant role.

Another game that is similar to a war of attrition is the sequential volunteer's dilemma studies by Bilodeau, Childs, and Mestelman (2004). In this game players have symmetric information about each other's bidding cost. In the only subgame perfect equilibrium of their game the bidding process ends in the first round with a bid of zero, i.e. in equilibrium one does not observe dynamically increasing bids. Bilodeau, Childs, and Mestelman observe substantially higher bids, i.e. again overbidding. We depart from Bilodeau, Childs, and Mestelman in analysing a situation with asymmetric information. The mutual bidding cost is only revealed at the end of the game, i.e. players do not know ex ante who is the weakest bidder.

Bidding procedure: Many other auction experiments use a static bidding procedure. In this paper we will compare two procedures: A static and a dynamic procedure.

With the static procedure players simply fill in a number on the screen and the computer immediately determines the winner and gains and losses from the game.

With the dynamic procedure participants see their bid and the time like an ascending clock on the screen and can, similar to the procedure in the Dutch auction, decide to stop the clock. Unlike in the Dutch auction, here the person who stops the clock is the loser of the auction. The explicit passing of time together with the visibly ascending cost might be an essential ingredient of the war of attrition that we want to study. With two bidders the two processes are strategically equivalent, but behaviourally there could be a difference. The evidence that we will present in section 4.4 confirms that, indeed, behaviour is not the same.

Uncertainty, number of contestants and prizes: Uncertainty about opponents' cost is essential for the war of attrition and first-price all-pay auction that we discussed in sections 2.1 and 2.2. We will have only two bidders and, therefore, only one prize.

Repetitions: The Bayesian Nash equilibria that we present in sections 2.1, 2.2, and 2.3 are the equilibria of a game that is played once or, by backward induction, equilibria of a game that is played finitely many times. We will study both situations: a sequence of one shot games with random matching of players after each round, but also games with random matching of players only after a fixed number of repetitions. While the one shot game constitutes a simple benchmark, we find the treatment with repetitions more interesting. Players who have fought over the allocation of a resource in the past will do this again. Regardless whether we interpret the war of attrition as an arms race, as the provision of public goods, the competition between firms, the settlement of strikes, or as political stabilisation—wars of attrition tend to repeat. Hence, most of our experiments are based on the finitely repeated war of attrition. Not only is the repeated situation closer to the conflict we actually want to model, studying repeated wars of attrition also has a technical advantage in the laboratory since the waiting time for other participants is considerably reduced.[6] Nevertheless, we study the one shot game with random rematching of players after each round in section 4.3.

[6]Since bids can have a large variance some wars of attrition will end quickly while others last for a long time. If players are rematched in each period, most players will have to wait most of the time. Otherwise these different bids average out which speeds up the experiment considerably.

number of sessions	number of subjects	$\underline{c}$	$\bar{c}$	repetitions in a group	static	firstprice
3	34	.38	3	6	0	0
3	36	1.8	3.6	6	0	0
3	36	2.2	4.4	1	0	0
4	46	2.2	4.4	6	0	0
3	50	2.2	4.4	6	1	0
3	44	2.2	4.4	6	1	1
3	36	2.6	5.2	6	0	0

TABLE 2: Overview over different treatments

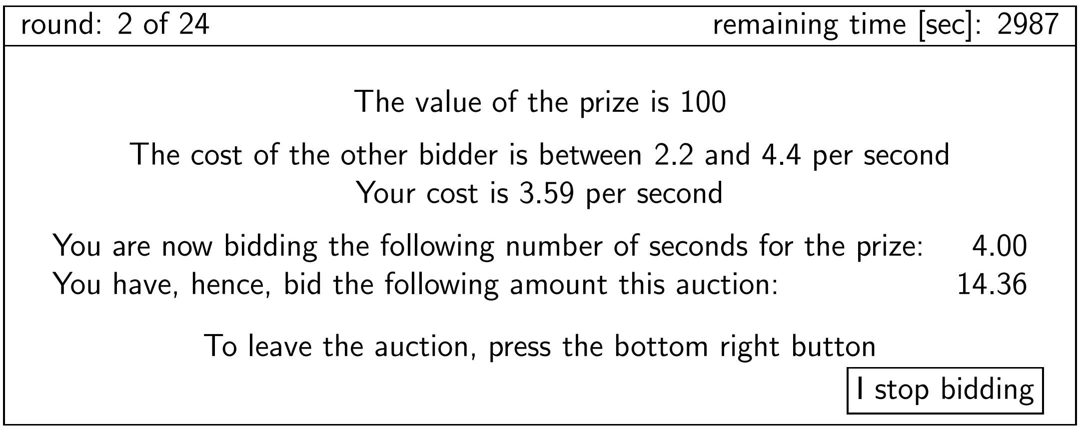


FIGURE 3: The bidding interface in the experiment

Our implementation All treatments of the experiment were implemented with the help of the software z-Tree (Fischbacher, 2007) and carried out at the experimental laboratory of the SFB 504 at the University of Mannheim. Table 2 lists the different treatments. Parameters for each session are given in section A of the appendix. We will describe the implementation of the dynamic bidding procedure here. The static procedure will be described in section 4.4 below.

In the treatments with dynamic bidding groups of typically 10 to 14 participants read instructions (see section B in the appendix), answer computerised control questions to check whether they understood the experiment, and are matched randomly in pairs to bid for a prize. During the bidding process participants see information similar to the one shown in figure 3. The number of seconds and the bid is updated

round: 2 of 24							remaining time [sec]: 17
The other bidder has won the auction							
auction	your cost per second	other bidder's cost per second	length of the auction in seconds	your cost (total)	other bidder's cost (total)	your profit in this auction	new balance of your account
2	3.59	2.91	4	14.36	11.64	-14.36	1876.54

FIGURE 4: Feedback given at the end of a round

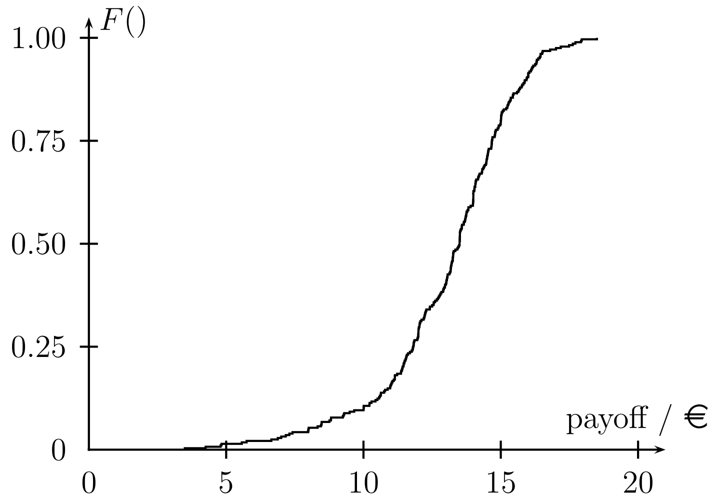
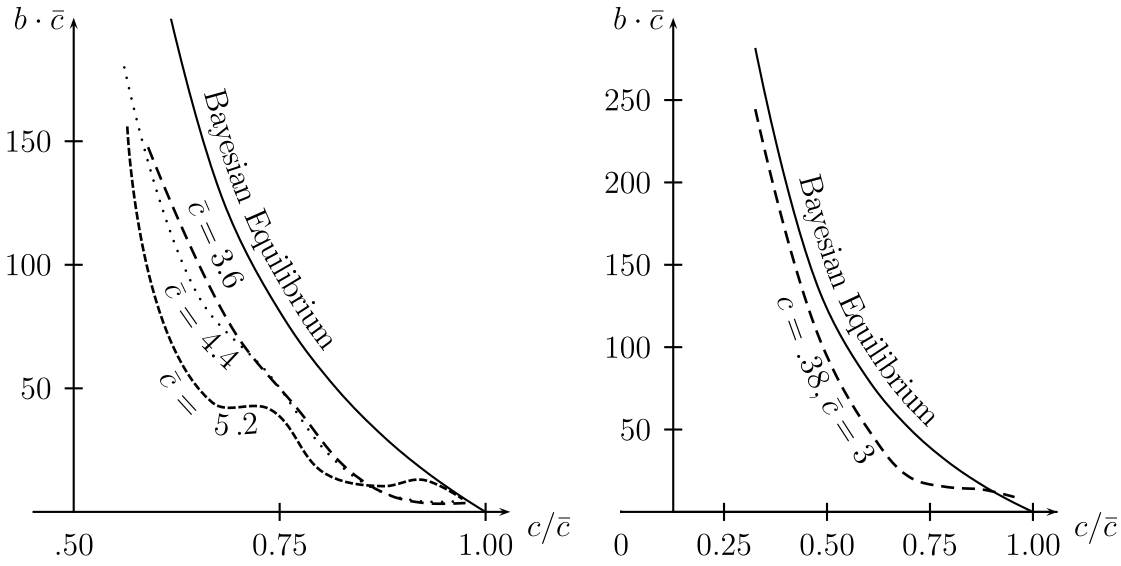


FIGURE 5: Cumulative distribution of total payoffs in the experiment

every second. As soon as one bidder stops bidding the other is declared winner of the auction. On the screen the participants get feedback similar to the one shown in figure 4. They are asked to copy this information manually to a table on their desk. Some of the feedback information, such as the other bidder's cost, might not always be available to bidders in a natural context. However, during out pilot experiments we saw that presenting the information in a symmetric way helps participants to understand the nature of the game.

We will first describe the treatment with repeated interaction. In this treatment participants play a sequence of six rounds with the same partner. Then they are matched again randomly for another six rounds. This procedure is repeated until the 24th round. At the end of the experiment participants complete a questionnaire, and receive their earnings from the experiment in sealed envelopes. Each session lasts for about 75 minutes. The cumulative distribution of payoffs at the end of the experiment is shown in figure 5.



Both figures shows median splines (7 bands) through the losers' bids. The left graph shows the three treatments where $\underline{c} = \bar{c}/2$. To show all three bidding functions in one graph the horizontal axis shows normalised cost $c/\bar{c}$ and the vertical axis shows normalised bids $b \cdot \bar{c}$. With this normalisation the equilibrium bidding function (solid line) is the same for all three treatments. The treatment with $\bar{c} = 3, \underline{c} = .38$ is shown on the right.
In all four treatments shown here bidders are matched for six periods and use the dynamic bidding procedure.

FIGURE 6: Bidding functions

4 Results

4.1 Bidding

Equation (4) describes equilibrium bids in the war of attrition. The higher the individual cost c_i the earlier a participant will give up. Figure 6 shows how the losing bid (in periods) depends on the parameters c_i and $\bar{c}$. The figure shows median splines through the losers' bids in the four treatments where bidders are matched for six periods repeatedly and use the dynamic bidding procedure. For the three treatments with $\underline{c} = \bar{c}/2$ we can normalise cost and bids such that the equilibrium bidding function (solid line) is the same for all three treatments. The lines below the solid lines show the losing bid in the experiment. We make the following observations:

- Median bids decrease with c_i. This is in line with the equilibrium bidding function.
- There is underbidding, i.e. bids in the experiment are below the equilibrium bidding function.

- For the three treatments where $\bar{c}/\underline{c} = 2$ the amount of underbidding seems to increase with $\bar{c}$, i.e. with the range of bidding costs.

While the first observation confirms our expectations, the second, namely that we find underbidding, is surprising in view of experiments with first-price all-pay auctions which consistently find overbidding. Anderson, Goeree, and Holt (1998) develop a model of boundedly rational bidders which relates overbidding to the number of bidders. The more bidders there are the more overbidding we should observe. Indeed, with two bidders we have the smallest number of bidders possible. Still, we find not only a small amount of overbidding, as would be consistent with Anderson, Goeree, and Holt (1998), but even underbidding.

While figure 6 gives a first impression, we have to carry out a more formal analysis in section 4.2 since here we only observe the loser's bid. If bidders deviate from their intended bids by small mistakes the above figure may provide a biased view of the intended bids.

4.2 Comparison with equilibrium bids

We use an interval regression to estimate individual bidding functions for each bidder i

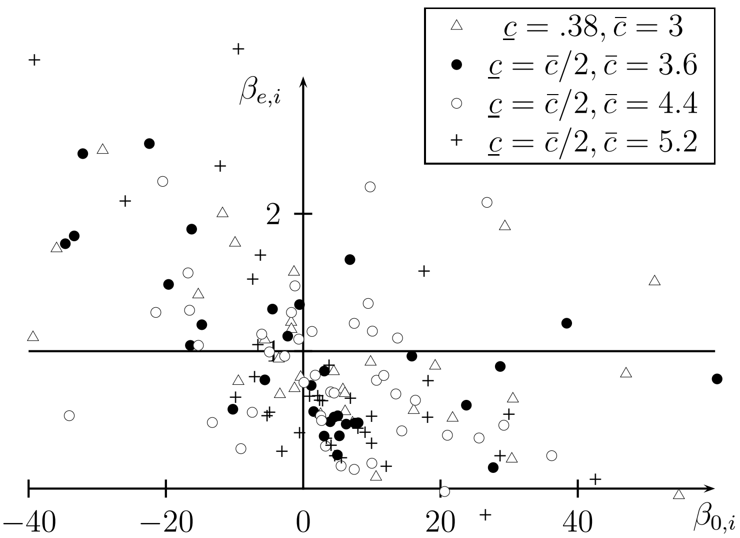
$$b_i = \beta_{e,i} \cdot \beta^W(c_i) + \beta_{0,i} + u \tag{15}$$

where $\beta^W(c_i)$ is the equilibrium bid from equation (4) and b_i is the actual bid. The parameters $\beta_{e,i}$ and $\beta_{0,i}$ are estimated separately for each individual. If all bidders follow the equilibrium bidding function we should find $\beta_{e,i} = 1$ and $\beta_{0,i} = 0$ for all bidders i. In each auction we observe a precise value for the loser's bid (the final bid, b_L) and an interval for the winner's bid (we know that $b_W \geq b_L$). Given this information the corresponding likelihood problem is maximised to estimate coefficients $\beta_{e,i}$ and $\beta_{0,i}$ (see Tobin, 1958; Amemiya, 1973, 1984).

A scatterplot of the individual estimates of equation (15) is shown in figure 7. The cumulative distribution of the individual estimates $\hat{\beta}_{e,i}$ is shown in figure 8. Two points are worth noting:

- The parameter $\beta_{e,i}$ is often smaller than one when it should be one in equilibrium. Most participants in our experiment react less sensitively to their bidding cost than they should in the risk neutral Bayesian equilibrium.[7] This

[7] We test whether the mean of the individual coefficients $\hat{\beta}_{e,i} = 1$. A parametric F-test (allowing for correlations within sessions) yields $F_{1,12} = 4.88$, $P_{>F} = 0.0473$, a non-parametric binomial test $P = 0.0159$.



Outliers have been eliminated using Hadi's method (Hadi, 1992, 1994).

FIGURE 7: Individual estimates of equation (15)

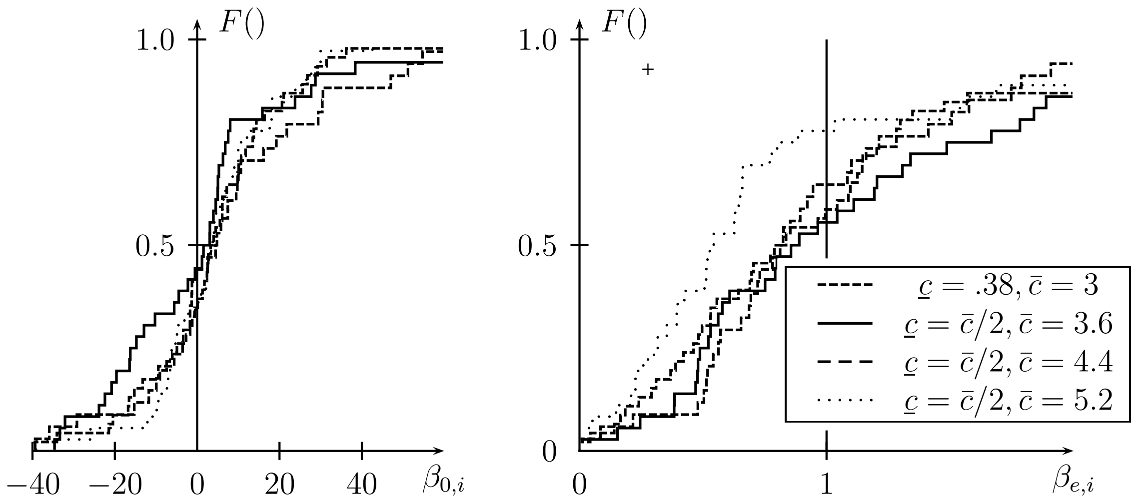
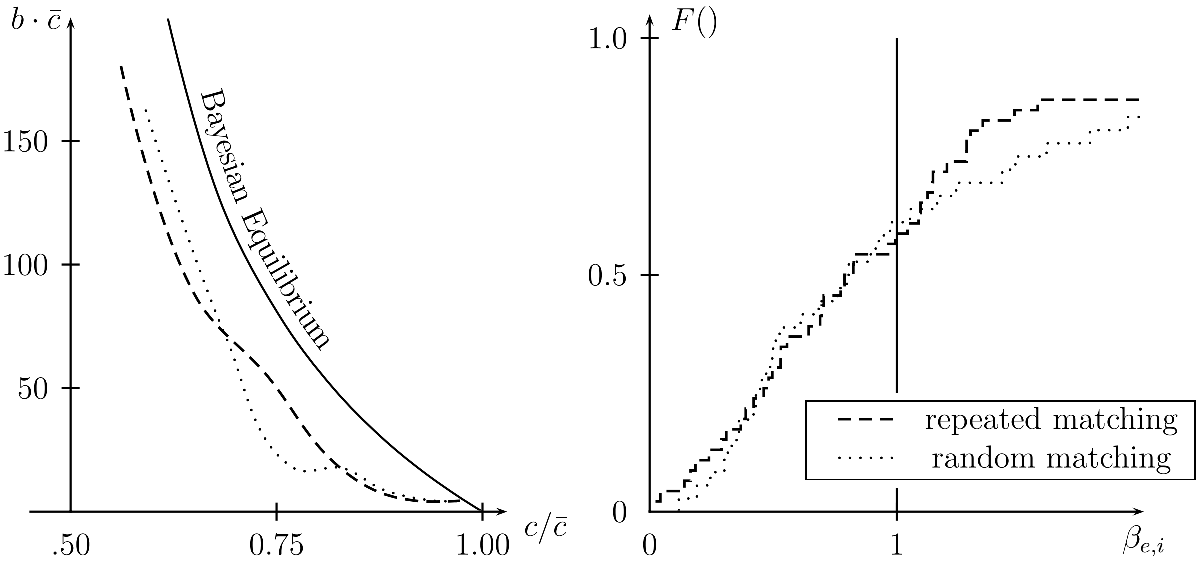


FIGURE 8: Cumulative distribution of $\beta_{0,i}$ and $\beta_{e,i}$ from equation (15)

is in contrast to most experiments with first-price all-pay auctions (see table 1).

- With larger $\bar{c}$ the sensitivity $\beta_{e,i}$ for c decreases. This trend, however, is just not significant.[8] Still, it is possible that the theoretical effect, namely that an increase in the range $[\underline{c}, \bar{c}]$ speeds up the end of the war, might be weakened, if not reversed.

[8] In a parametric test we regress $\beta_{e,i}$ on $\bar{c}$ and test wether the coefficient is zero. A t-test finds t=-1.46, $P_{>|t|} = 0.169$. This test (as well as all other tests in this paper) takes into account that observations within an experimental session may be correlated by using the procedure of Rogers (1993). In a nonparametric Cuzick-Altman we test whether $\beta_{e,i}$ follows a trend over $\bar{c}$ and find $z = 1.58$, $P_{>|z|} = 0.11$.



The figure compares bidding with repeated and with random matching. In the left part of the figure we show median bids of the losers (similar to figure 6), in the right graph we show the cumulative distribution of $\hat{\beta}_{e,i}$ (similar to figure 8). In both graphs we show only the case $\underline{c} = 2.2, \bar{c} = 4.4$.

Figure 9: Collusion

4.3 Collusion and Repetition

In the previous sections we have seen that players bid less in the experiment than what they would bid in the risk neutral Bayesian equilibrium. A possible explanation could be collusive behaviour. To measure the degree of collusive behaviour due to the repeated matching we ran one treatment where players are rematched randomly after each auction (and not only every six auctions as in all other treatments). In this treatment $\underline{c} = 2.2$ and $\bar{c} = 4.4$. Figure 9 shows for this case the median spline through the bidding function and the distribution of $\hat{\beta}_{e,i}$ for two treatments: One where players are repeatedly matched for six periods and another where players are randomly rematched after each interaction. We see that there is no difference between the two treatments. In particular in both treatments most players have a $\hat{\beta}_{e,i} < 1$, i.e. less than the equilibrium value.[9]

[9]The difference between the two treatments is not significant. A Two-sample Wilcoxon rank-sum (Mann-Whitney) test yields $z = -0.707$, $P_{>|z|} = 0.4795$. A parametric t-test yields $t = -.53$, $P_{>|t|} = .609$. The results of estimating equation (17) in section 4.4 are in line with this observation.

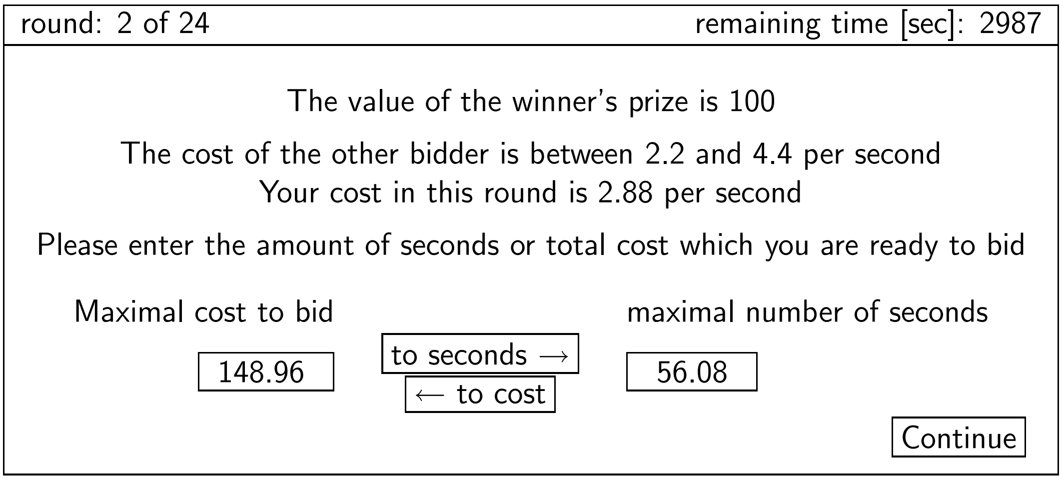


FIGURE 10: The interface in the static treatment

4.4 Static and dynamic bidding

Let us come back to experimental evidence for first-price all-pay auctions in the literature.[10] All these experiments find overbidding in first-price all-pay auctions. Our setup seems to be similar, still we find underbidding in our experiment. To understand why this might be the case let us look at table 1 where we list some parameters of our experiments and of the experiments in the literature. While most experiments with first-price all-pay auctions in the literature use a static procedure the results reported for the war of attrition in sections 4.1 to 4.3 refer to a dynamic procedure.

To control for that parameter we also did, in addition to the dynamic war of attrition, experiments with a static war of attrition and a static first-price all-pay auction.[11] A typical screen for the static procedure looks like the one in figure 10. In contrast to the dynamic procedure (figure 3) players make decisions before the actual bidding begins. They either fix a highest cost or a highest number of seconds up to which they are willing to bid. The interface in the experiment automatically converts cost into seconds and vice versa. Once players have made their choice the actual bidding process completes instantaneously.

Let us call the equilibrium bid $\beta^E \equiv \beta^W(c_i)d^W + \beta^A(c_i)d^A$ where d^W is a dummy that is one in the war of attrition and d^A is a dummy that is one in the first-price all-pay auction. The functions $\beta^W(c_i)$ and $\beta^A(c_i)$ denote the equilibrium bids in the

[10] Potters, de Vries, and van Winden (1998), Davis and Reilly (1998), Barut, Kovenock, and Noussair (2002), Müller and Schotter (2007).

[11] In the list of experiments in appendix A the sessions with *static*=1 use the static bidding procedure. The sessions where *firstprice*=1 are first-price all-pay auctions. We study only $\underline{c} = 2.2$, $\bar{c} = 4.4$ in the static case and the first-price all-pay auction.

	β	σ	t	$P_{>\|t\|}$	95%conf. interval
β_{asc}	$-.18279$	.05572	-3.281	0.007	$-.30419, -.0614$
β_{stat}	$-.07889$	.07118	-1.108	0.289	$-.23398, .0762$
β_{first}	.13379	.08353	1.602	0.135	$-.04821, .3158$
robust regression, 13 independent obs.					

We use the method of Rogers (1993) to account for correlations of observations within sessions.

Table 3: Bidding functions: Results of estimating equation (17)

war of attrition and first-price all-pay auction respectively (see equations (4) and (9)). For all three treatments we estimate for each bidder i individual coefficient $\beta_{e,i}$ for the following equation[12]

$$b_i = \beta_{e,i} \cdot \beta^E + u \tag{16}$$

where b_i is the actual bid and β^E the equilibrium bid. To be consistent we use only the data from the treatments where $\underline{c} = 2.2$ and $\bar{c} = 4.4$ (see table 2). Furthermore, we use the interval regression method for all treatments. Hence, in the static and first-price all-pay auction treatments where we can actually observe the winner's bid and where we could also use OLS, we only use the loser's bid that can be observed in all treatments. For the winner of the auction we use only the information that the winner's bid must be larger than the loser's bid. Thus, even if the interval regression would introduce some bias, we have the same bias in all treatments. If bidders follow the Bayesian Nash equilibrium bidding function then $\beta_{e,i} = 1$. To test this, we estimate

$$\hat{\beta}_{e,i} - 1 = \beta_{\text{asc}} d_{\text{asc}} + \beta_{\text{stat}} d_{\text{stat}} + \beta_{\text{first}} d_{\text{first}} + u \tag{17}$$

where $\hat{\beta}_{e,i}$ is the individually estimated coefficient from equation (16), d_{asc} is a dummy that is one for the dynamic war of attrition treatment, d_{stat} is a dummy that is one for the static war of attrition treatment, and d_{first} is a dummy that is one for the first-price all-pay auction treatment. The coefficients that we estimate are β_{asc}, β_{stat}, and β_{first}. If bidders follow the Bayesian Nash equilibrium bidding function then $\beta_{\text{asc}} = \beta_{\text{stat}} = \beta_{\text{first}} = 0$. Results of the estimation using the interval regression method are shown in table 3.[13]

We see that β_{asc} is significantly smaller than zero. This confirms what we already

[12] We estimated several variants of this equation, including one that includes a constant similar to equation (15). This does not change any of the results.

[13] Outliers have been eliminated using Hadi's method (Hadi, 1992, 1994).

know: there is underbidding in the ascending clock treatment in the war of attrition.

The estimate for β_{stat} is still negative, but not significant. There is still some underbidding (though less so) in the static treatment in the war of attrition.

The estimate of β_{first} is positive, thus, we find some, though not significant, overbidding in the first-price all-pay auction treatment. This confirms what we should expect from the literature on first-price all-pay auctions.

4.5 Revenue comparison

Following equations (5) and (10) we should expect the same revenue for the war of attrition and the first-price all-pay auction. For other distributions of the bidding cost this need not be the case. One can show that the expected revenue in the war of attrition is never smaller than in the first-price all-pay auction, it can only be larger (see Krishna and Morgan, 1997). However, if in the experiment bids are smaller than equilibrium in the war of attrition and larger in the first-price all-pay auction what can we say about revenue? Average overbidding in the first-price all-pay auction does not necessarily imply higher revenues, at least not if the amount of overbidding depends on the bidding cost. Similarly, average underbidding in the war of attrition does not need to imply lower than equilibrium revenues.

To compare revenues in the experiment we estimate the following equation

$$R = R^E \cdot (1 + \beta_{\text{asc}} d_{\text{asc}} + \beta_{\text{stat}} d_{\text{stat}} + \beta_{\text{first}} d_{\text{first}}) + u \tag{18}$$

where R is the actual revenue obtained in the auction and R^E the revenue in equilibrium given by equations (5) and (10). The dummies d_{asc}, d_{stat}, and d_{first} have the same interpretation as in equation (17). If all bidders follow the Bayesian Nash equilibrium bidding function we should have $\beta_{\text{asc}} = \beta_{\text{first}} = \beta_{\text{stat}} = 0$. If the coefficients are negative then the revenue is smaller than in equilibrium, if the coefficients are positive then the revenue is larger than in equilibrium. Results of the estimation are shown in table 4.[14] All coefficients are negative, however, β_{first} only by a small amount and not significantly. Thus, revenue is significantly smaller than equilibrium revenue only in the war of attrition, not in the first-price all-pay auction.

[14]Outliers have been eliminated using Hadi's method (Hadi, 1992, 1994).

	β	σ	t	$P_{>\|t\|}$	95%conf. interval
β_{asc}	$-.20972$	.06974	-3.007	0.011	$-.36166, -.05778$
β_{stat}	$-.11367$	.03406	-3.338	0.006	$-.18787, -.03947$
β_{first}	$-.04346$	.03749	-1.159	0.269	$-.12514, .03822$
robust regression, 13 independent obs.					

We use the method of Rogers (1993) to account for correlations of observations within sessions.

Table 4: Revenue: Results of estimating equation (18)

$\bar{c}$		$\bar{\beta}_{p,i}$	t	$P_{>\|t\|}$
3	dynamic war of attrition	$-.00824$	$-.53454$	.64644
3.6	dynamic war of attrition	.00481	.79784	.50864
4.4	dynamic war of attrition	$-.01051$	-1.7669	.17542
5.2	dynamic war of attrition	$-.00688$	-2.1889	.16006
4.4	random matching dynamic war of attrition	.00468	1.2255	.34511
4.4	static war of attrition	$-.00055$	$-.05563$	.9607
4.4	first-price all-pay auction	$-.00578$	-1.267	.33271

The table shows estimates of $\beta_{p,i}$ from equation (19). We use the method of Rogers (1993) to account for correlations of observations within sessions.

Table 5: Learning

4.6 Learning

To see whether players change their behaviour during the experiment and perhaps converge to the Bayesian Nash equilibrium we extend equation (16) by a term that allows bidding behaviour to change over time

$$b_i = (\beta_{e,i} + \beta_{p,i} \cdot p) \cdot \beta^E + \beta_{0,i} + u \tag{19}$$

where p is the period in the experiment. If behaviour does not change over time $\beta_{p,i}$ should be zero, if bids increase $\beta_{p,i}$ should be positive etc.[15] If bids converge to equilibrium values we should, given the estimation results for equation (17) reported in table 3, expect a positive $\beta_{p,i}$ in the dynamic war of attrition and a negative $\beta_{p,i}$ in the first-price all-pay auction. Table 5 shows average estimated values for $\beta_{p,i}$ for the different treatments and reports tests whether $\beta_{p,i} = 0$.[16] None of the coefficients is significantly different from zero.

[15]The specification that we use here measures the change in bidding behaviour relative to $\beta^W(c_i)$. One can do the same exercise with absolute changes and obtains similar results.

[16]Outliers have been eliminated using Hadi's method (Hadi, 1992, 1994).

4.7 Bifurcations in first-price all-pay auctions and in wars of attrition

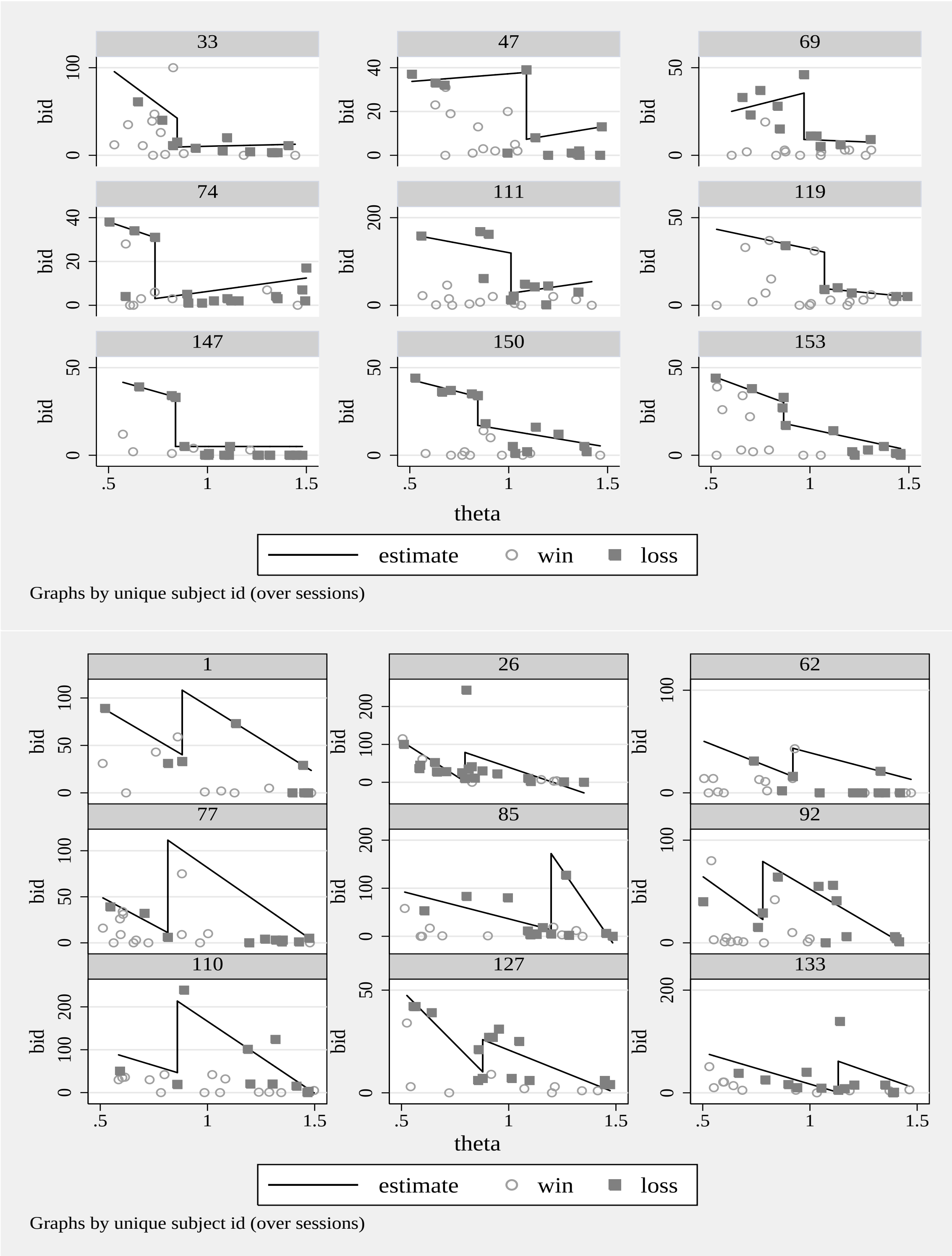
In an experiment with first-price all-pay auctions Müller and Schotter (2007) observe what they call bifurcations of bidding functions. If bidding is expensive, bidders underbid, if bidding is cheap, bidders overbid (see figure 2 on page 9). We want to find out whether this is a stable pattern that also repeats in our experiments with wars of attrition. To test this, we will first follow the procedure suggested by Müller and Schotter. We estimate the following switching regression:

$$b_i(\theta) = \begin{cases} \beta_{e,i}\theta + \beta_{0,i} + u & \text{if } \theta \leq \hat{\theta}_i \\ \gamma_{e,i}\theta + \gamma_{0,i} + u & \text{if } \theta > \hat{\theta}_i \end{cases} \tag{20}$$

We can use an OLS regression for the static war of attrition and the first-price all-pay auction. For the dynamic war of attrition we do not observe the winner's bid, and, thus, have to use an interval regression (Tobin, 1958; Amemiya, 1973, 1984). For each individual i we estimate coefficients $\hat{\beta}_{e,i}, \hat{\beta}_{0,i}, \hat{\gamma}_{e,i}, \hat{\gamma}_{0,i}$. Also for each individual we choose the position of the step $\hat{\theta}_i$ such that the likelihood of the interval regression is maximised.[17] Figure 11 shows several examples of estimated individual bidding functions. The upper part of the figure shows estimated bidding functions which correspond to the description of Müller and Schotter. For low cost, the bidding is relatively high, at the threshold value there is a drop to a segment that is lower. However, there are also other participants which are described by bidding functions like the ones in lower part of figure 11. These participants do not show a drop in the bidding function at the threshold, but, instead, an increase. Müller and Schotter suggest the following test for what they call the bifurcation hypothesis: If the residual sum of squares RSS_{SR} of the switching regression (20) is smaller than the residual sum of squares RSS_{ALT} of the alternative model from equation (21) (which contains the equilibrium bidding function as a special case), then the switching regression model has a better fit, and is, thus, according to Müller and Schotter, supported.

$$b_{it} = \beta_0 + \beta_1 c_{it} + \beta_2 c_{it}^2 + \beta_3 \ln c_{it} + u \tag{21}$$

[17]For the interval regression we do not obtain convergence of the estimator for 2 of our 282 participants. To check overall convergence we did all our estimates twice, once with at most five iterations, and once with at most 25 iterations. The results are practically the same (we have looked at distributions of estimated coefficients like those presented in figure 12 and found that they are visually indistinguishable for 5 and for 25 iterations) so that we have no reason to believe that a larger number of iterations might change the results.



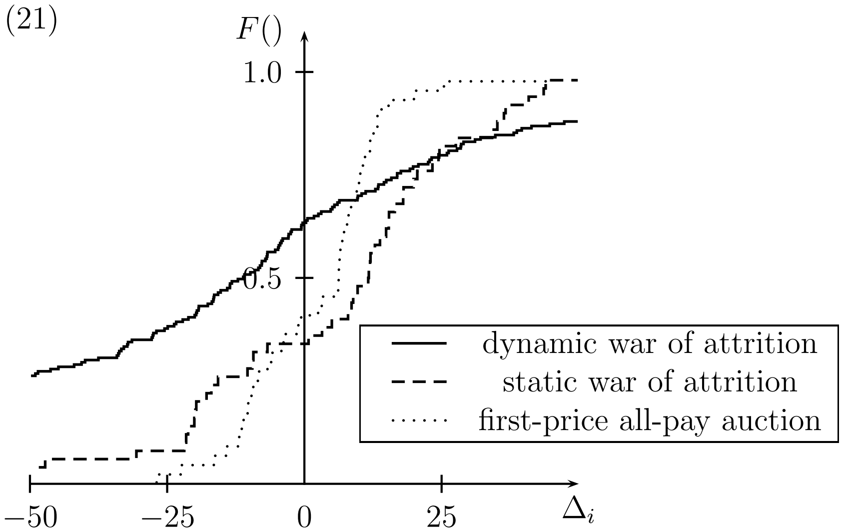
Actual bidding is approximated with stepwise linear functions. Some bidding functions descend from the left to right (as the ones in the top graph), but others have an ascending step (as the ones in the bottom graph). The ones in the top graph are consistent with Müller and Schotter's findings, the others are not.

FIGURE 11: Bifurcated bidding functions for 18 participants in our experiment

	n	$\text{RSS}_{\text{SR}} - \text{RSS}_{\text{ALT}}$	t	$P_{>\|t\|}$	P_{bin}
static war of attrition	3	-3431	-1.280	0.329	0.250
first-price all-pay auction	3	-99.94	-6.757	0.021	0.250

Since bidding functions of the dynamic war of attrition can not be estimated with the help of OLS, no results for the dynamic war of attrition are reported here.

TABLE 6: Test for the difference in the residual sum of squares of OLS estimates of equations (20) and (21)



The graph shows the cumulative distribution of the step size. Only positive step sizes are consistent with Müller and Schotter's workaholics and drop outs. For the static treatments Δ_i is given for an OLS approach, for the dynamic treatment Δ_i is shown for the interval regression.

FIGURE 12: Bifurcation, the distribution of Δ_i

Test statistics for the difference $\text{RSS}_{\text{SR}} - \text{RSS}_{\text{ALT}}$ in our experiment are reported in table 6. Indeed, as in Müller and Schotter, also in our experiment the difference $\text{RSS}_{\text{SR}} - \text{RSS}_{\text{ALT}}$ is significantly negative for the first-price all-pay auction, i.e. the switching regression model gives a better fit. This is where Müller and Schotter (2007) stop and conclude they found support for the switching regression model.

In the next paragraph we suggest a different test. We will study the step size in the switching regression model, i.e., the difference

$$\Delta_i = \hat{\beta}_{e,i}\hat{\theta}_i + \hat{\beta}_{0,i} - (\hat{\gamma}_{e,i}\hat{\theta}_i + \hat{\gamma}_{0,i}) \tag{22}$$

This difference is positive if participants have a decreasing step in their bidding function like in the upper part on figure 11. Figure 12 shows the distribution of the step size. Let us first look at the dotted line which shows the distribution of the step size Δ_i for the first-price all-pay auction which is the auction format which has also

	n	$\bar{\Delta}_i$	t	$P_{>\|t\|}$	P_{bin}
dynamic war of attrition (interval regression)	16	-17.09	-4.198	0.001	0.077
static war of attrition (OLS)	3	-.0043	-0.001	0.999	1.000
static first-price all-pay auction (OLS)	3	2.474	1.521	0.268	1.000

TABLE 7: Two-sided tests against $\Delta_i = 0$

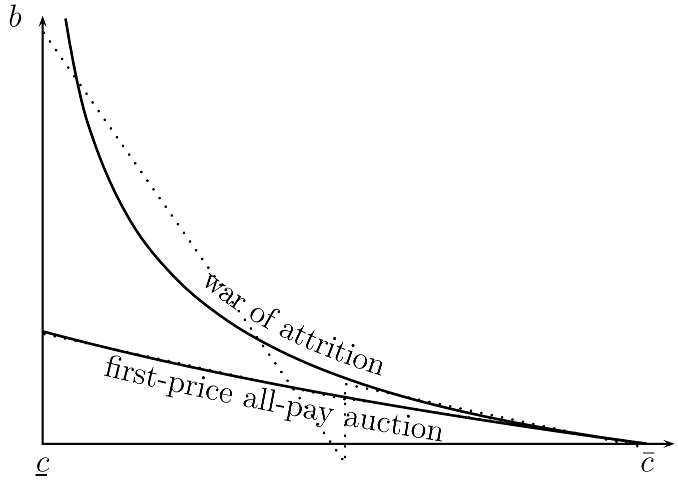


FIGURE 13: Approximating equilibrium bidding functions with stepwise linear functions

been studied by Müller and Schotter. Indeed, more than 50% have an estimated positive stepsize, though, not many more. There seems to be an almost equally large fraction of players with a negative stepsize.

For the different treatments we test against $\Delta_i = 0$ and report results in table 7. For the first-price all-pay auction and the static war of attrition we do not find Δ_i to be significantly different from zero. We see that Δ_i is actually significantly negative in the dynamic war of attrition.

Figure 12 and the test results reported in table 7 might suggest that, after all, there are not so many workaholics in the experiment and that the switching regression may rather pick up some noise. The smaller RSS of equation (20) might be a result of the rather collinear regressors in equation (21) which have been chosen by Müller and Schotter.

How can it be that in our experiment we find a negative stepsize for the dynamic war of attrition treatment? To understand this better let us compare the equilibrium bidding functions in the war of attrition and in the first-price all-pay auction. Figure

13 shows examples for the equilibrium bidding functions in the war of attrition and the first-price all-pay auction as solid lines. Since $\lim_{c \to \underline{c}} \beta^W(c) = \infty$ the function is not easy to approximate with a stepwise linear function. The dotted line shows the result of an OLS regression of a random sample of cost values. We see that, since the left part of the function is very steep, we obtain a negative stepsize.

The equilibrium bidding function for the all-pay auction (equation (9)) is much easier to fit with a stepwise linear function (the dotted line is very close to the equilibrium bidding function). Any step of a stepwise linear approximation will be small.

5 Concluding remarks

In this series of experiments we address a couple of hypotheses, some related to theoretical studies of the war of attrition, some to other experiments. Some of our observations are in line with theory. Basic properties, like the one that with higher bidding cost bids decrease, can be confirmed. However, there are some significant deviations from the theory. Some of these deviations are in line with what we should expect from other experiments, others are not.

Other experiments with first-price all-pay auctions find bids which are higher in the laboratory than they are in the risk neutral Bayesian equilibrium. In our setup such a deviation can not be explained with risk aversion, and, indeed, in our experiment we have seen that this overbidding seem to be a specific property of the first-price format which can not be generalised to wars of attrition, in particular not for the case of a dynamic bidding procedure.

Theory would predict that expected revenue is, if at all, higher in the war of attrition than it is in the first-price all-pay auction if bidders are risk neutral. However, if bidders are risk averse, then revenue may well be higher in the first-price all-pay auction than in the war of attrition — which is what we have found in our experiment.

From the findings of Müller and Schotter (2007) we should expect discontinuous individual bidding functions and what they call a 'bifurcation of effort', i.e. a bidding function with a downward leading step where bidders with a high cost make small or no bids at all (these are the 'drop outs' in Müller and Schotter's story) while bidders with a small cost bid too much (the 'workaholics'). While we can find support for the results of Müller and Schotter (2007) in their experimental context, we have seen that the method they are using (in particular comparing the RSS of equations

(20) and (21) should not be over-interpreted. The distribution of the step size Δ_i might indicate that there are not so many workaholics after all. In any case, the majority of workaholics disappears in wars of attrition. At that stage we should ask ourselves: what is the appropriate model for competition at the workplace. As long as workers can see their mutual efforts (which seems to be the case at many workplaces) the winning worker only has to make a slightly higher effort than the losers, thus, effectively these workers are playing a second-price all pay auction, a war of attrition. And in a war of attrition we did not really find workaholics.

While our study answers some questions it also opens a couple of new ones. We do not know why the dynamic war of attrition leads to more underbidding and less revenue than the static war of attrition. Deviations from equilibrium bidding in the Dutch auction is sometimes related to false updating which might play here a role as well. In any case, our experiment shows that it is possible to study stabilisation processes and other wars of attrition in the lab and opens some room for further research.

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A List of experiments

	Date	$\underline{c}$	$\bar{c}$	repetitions in a group	static	firstprice	periods	subjects
1	20040116-10:39	2.2	4.4	6	0	0	18	10
2	20040116-14:09	2.2	4.4	6	0	0	18	10
3	20040120-10:31	2.2	4.4	6	0	0	24	14
4	20040120-12:25	2.2	4.4	6	0	0	24	12
5	20040120-13:57	2.6	5.2	6	0	0	24	12
6	20040122-14:01	2.6	5.2	6	0	0	24	12
7	20040122-15:53	2.6	5.2	6	0	0	24	12
8	20040123-10:31	1.8	3.6	6	0	0	24	12
9	20040123-12:19	1.8	3.6	6	0	0	24	12
10	20040123-16:07	1.8	3.6	6	0	0	24	12
11	20040219-10:31	2.2	4.4	1	0	0	24	12
12	20040219-13:35	2.2	4.4	1	0	0	24	12
13	20040219-15:49	2.2	4.4	1	0	0	24	12
14	20040220-10:23	.38	3	6	0	0	24	8
15	20040220-12:25	.38	3	6	0	0	24	12
16	20040220-16:17	.38	3	6	0	0	24	14
17	20040524-15:17	2.2	4.4	6	1	0	24	16
18	20040525-14:01	2.2	4.4	6	1	0	24	16
19	20040525-16:07	2.2	4.4	6	1	0	24	18
20	20041021-15:59	2.2	4.4	6	1	1	24	16
21	20041022-10:33	2.2	4.4	6	1	1	24	14
22	20041027-15:49	2.2	4.4	6	1	1	24	14

The experiment was carried out in the experimental laboratory of the SFB 504 at the University of Mannheim. All sessions were conducted in German. Section B contains a translation of the instructions.

B Conducting the experiment and instructions

Participants were recruited by email and could register for the experiment on the internet. At the beginning of the experiment participants were randomly allocated to seats and obtained printed instructions in German. A translation can be found below.

After reading the instructions participants start with control questions on the screen, then go through the actual treatment, conclude with a short questionnaire on the screen and are payed in cash immediately after the experiment. The experimental software is based on z-Tree Fischbacher (2007).

Translation of the instructions:

Welcome to a strategy experiment

This strategy experiment is financed by the University of Mannheim and the Deutsche Forschungsgemeinschaft (DFG). The instructions are easy to understand when you read them carefully. If you decide considerately and take into account the position of the other players you have the opportunity to gain a considerable amount of money. You receive the money at the end of the game. The profit is related to your performance during the game.

During the experiment you participate in an auction about prizes which are valued in "Experimental Currency Units" (ECU). During the auction your bids are also in ECU. At the end of the auction you will be paid in Euro. Thereby, 200 ECU equal 1 Euro. We have already held experiments similar to this one. Due to our experience we expect an average profit of 12 Euro, dependent on your strategy. We have no interest in paying you less money than you are entitled to. The amount of money not used will be returned to the Deutsche Forschungsgemeinschaft (DFG).

During the experiment talking and communicating between the bidders is strongly prohibited. Your are not allowed to take any notes, books, and cell phones into the experimental laboratory. Moreover, you are not allowed to start other programs on the computers. If you don't follow the rules we have to exclude you from the experiment and you won't get any payment.

Instructions You play an auction which is held between two bidders. Bidders are randomly and anonymously assigned to each other. Each pair of bidders play 6 rounds together. Four times during the experiment you get an new partner, randomly selected.[18] In total you will play 24 rounds, thereof each 6 sequenced rounds with the same partner. Each single round corresponds to one auction in which one prize is sold. The value of the auction prize is in all 24 rounds and for all participants 100 ECU. Once an auction has started you and the other bidder pay in every second a certain amount until either you or the other bidder are not willing to increase the bid.

In the beginning of each auction you will be informed of your bidding cost per second which corresponds to the amount of ECU you bid every second. During each auction the bidding cost per second are constant. At the beginning of every round each bidder randomly receives new bidding cost per second to participate in the auction of the prize. The bidding cost per second for both bidders are uniformly distributed between 2.20 and 4.40 ECU[19] but you have no information about the

[18] *In the treatment where players are rematched after each round the instructions were:* "The bidders are randomly and anonymously assigned to each other in each round of the experiment. In total you will play 24 rounds. Each single round corresponds to one auction in which one prize is sold."

[19] *The cost varies from treatment to treatment.*

exact bidding cost per second of your partner.[20]

round: 2 of 24	remaining time [sec]: 2987

The value of the prize is 100

The cost of the other bidder is between 2.2 and 4.4 per second
Your cost is 3.59 per second

You are now bidding the following number of seconds for the prize:	4.00
You have, hence, bid the following amount this auction:	14.34

To leave the auction, press the bottom right button

I stop bidding

Each period, the screen shows the value of the prize which is constantly 100 ECU. Furthermore, the screen displays your bidding cost per second and reminds you that the bidding cost per second is an amount between 2.20 and 4.40.

The next part of the instructions differs between the dynamic war of attrition treatment and the static war of attrition and first-price all-pay auction treatment. The instructions in the dynamic treatment were as follows:

> In addition, you will be informed about how many seconds you have already bid and what your total cost of the current auction are. After 10 seconds a “Stop”-button with the title ”I stop bidding” appears down right. You should use the countdown (10 seconds) to plan your optimal bidding strategy. Press the “Stop”-button if you don’t want to proceed bidding and leave the auction. As soon as you leave the auction your partner wins the prize. Likewise, you win the prize if your partner leaves the auction earlier as you. **As long as you don’t press the “Stop”-button, you are still bidding for the prize. The auction ends for both bidders as soon as the first bidder presses the “Stop”-button.**
>
> **For every second you bid, you have to pay the bidding cost per second. These cost occur independently of who (you or your partner) wins the auction.**

The instructions in the static and first-price treatment were as follows

> You can enter how many seconds or up to which amount of total cost you a ready to continue bidding. You can click on either the “to seconds” translate cost into seconds or the “to cost” button to translate seconds into

[20] *In the static treatment and in the first-price treatment we did not show the following picture but instead the interface from figure 10.*

cost. You can repeat this as often as you like until you are ready to commit to your bidding strategy, i.e. the amount of seconds up to which you want to bid or the cost up to which you want to bid.

Please note that some participants have the input field for cost on the left side of the screen and for seconds on the right side of the screen while others have the opposite layout.

In the lower right corner of the screen you find a button "Continue". You have to push this button once you have determined your bid. If the other bidder has chosen a higher number of seconds, he wins the prize. If the other bidder has chosen a smaller number of seconds, you win the prize. If both bidders have chosen the same number of seconds then the winner is determined randomly.

With your bid in seconds you determine how many seconds you continue to bid at most. The bidder with the smaller amount of seconds determines the end of the auction. The number of seconds he chose will be used to determine the cost for both bidders. Up to this time both bidders pay for each second their cost. The bidder whose bid in seconds is larger obtains the prize of 100 ECU.[21]

Note that for each second that you are bidding for the prize you have to pay your bidding cost per second — independently whether you or the other bidder wins the prize at the end of the auction.

From here on the instructions were again the same in both treatments:

In the beginning of the experiment your account balance is 2500 ECU. Your cost will be subtracted from the account balance. If you win the auction, the prize with the value of 100 ECU will be credited to your account. Your account balance at the end of each auction is calculated as follows:

	Account balance before the start of the auction
−	(Number of bidding seconds) × (Bidding cost per second)
+	Value of the prize, if you win the auction
=	Account balance after the auction

The account balance at the end of the auction is your account balance at the beginning of the following auction. The account balance at the end of the 24th auction is your payoff for the participation of the experiment. Thereby, you receive 1 Euro for 200 ECU.

At the end of each auction both bidders will be informed about their bid in seconds, total bidding cost, current account balance and who has won the prize. Furthermore,

[21] *In the first-price treatment this text would read as follows:* "With your bid in seconds you determine how many seconds you continue to bid. The number of seconds each bidder chooses will be used to determine his cost. The bidder whose bid in seconds is larger obtains the prize of 100 ECU."

each bidder gets the information about his partner's bidding cost per second and total bidding cost in the previous auction.

During the experiment please fill in the table below at the end of each auction. Then you always know the bidding cost per second and the total cost of you and your partner and you have an overview of your gains and the development of your personal account.

If you have any questions, please don't hesitate to rise your hand. We will be glad to come to your seat and answer your questions.

Thank you very much for your participation!

Participants would find a table like one of the following at their desk. The were asked to fill in the result of each round into the table. This information was also shown on the screen as a feedback for each round (see figure 4).

dynamic treatment:

Auction	your cost per second	other bidder's cost per second	length of the auction in seconds	your cost (total)	other bidder's cost (total)	winner in the auction	your profit in this auction	new balance of your account
1								
2								
⋮								

Static treatment:

Auction	your cost per second	other bidder's cost per second	you bid at most … seconds	you bid at most a cost of …	length of the auction in seconds	your cost (total)	other bidder's cost (total)	winner in the auction	your profit in this auction	new balance of your account
1										
2										
⋮										

First-price treatment:

Auction	your cost per second	you bid … seconds	you bid a cost of…	other bidder's cost per second	the other bids …seconds	the other bids a cost of…	winner in the auction	your profit in this auction	new balance of your account
1									
2									
⋮									