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Exploration and Exploitation – The Role of Entrepreneurship and R&D in the Process of Innovation

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Abstract

We formulate a model that explicitly separates two functions in the innovation process: The introduction of new goods and the quality improvement of existing goods. While the latter is performed by the corporate R&D sector, the first is performed by entrepreneurs. We show that in a three sector economy, which also includes a producing sector, there exists a stable non trivial allocation of labor to production, innovation and entrepreneurship. We compute the steady state allocation of labor to production, R&D and Entrepreneurship. We show that the innovation rate decreases if one of the innovative sectors does not exist.

JEL-classification: O31, O41

Keywords: Innovation, Variety Expansion, Quality Ladders, Entrepreneurship, R&D Sector.

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It has long been recognized that the
entrepreneurial function is a vital component
in the process of economic growth.
William J. Baumol (1968, p. 65)

1 Introduction

Research on innovation and its role in economic growth has by now a long tradition in economic research. In formalized models of the interaction between innovation on economic growth, it is possible to distinguish two different approaches. One approach considers innovation as an *expansion of the number* of available technologies in the producing sector or in the intermediate goods sector. In these models, human capital is explicitly devoted to expand the knowledge base, thus increasing total factor productivity and subsequently economic growth. Key references to this kind of models are of course Romer (1990) and Lucas (1988), together with Grossman and Helpman (1991a).

Another set of models considers innovation as an *improvement of quality* of a given and constant number of existing products or technologies. In these models, R&D is the driving engine behind the quality improvement, resulting in a stochastic progression of quality of these goods. While this process generates a negative externality for existing qualities, it generates a constant growth rate of utility for consumers. Key references for this type of growth theory are Grossman and Helpman (1991b) or Aghion and Howitt (1992). Grossman and Helpman (1991b, Section IV) show that both approaches can lead to similar formalizations hence yield similar results as to the growth rate of utility.

The dichotomy of variety expansion and quality improvement corresponds to another dichotomy in the innovation process that is discussed under the notions of *exploration* versus *exploitation*. March (1991) argues that "explorative search" consists in experimenting with new options from which new possibilities can be learned from, "exploitative search" consists in the identification, routinization, and extension of good ideas. While March discussed these two types of search in the context of organizational learning, they can be applied in a framework of innovation research. In this analogy, exploration of new technologies corresponds to variety expansion while exploitation of existing technologies corresponds to quality improvement.

A number of empirical studies confirm the hypothesis of the existence of these two different approaches to developing newness (Acs and Audretsch, 1991; Audretsch, 1991; Breschi, Malerba and Orsenigo, 2000). However, while it is conceptually useful to distinguish between both types of innovation, they usually come together in the process of development and diffusion of new technologies. At the same time, both types of innovation are executed by different agents. Almeida and Kogut (1997) and Almeida (1999) argue that usually small firms, are more likely to

explore new and unknown technologies, leaving the development of more mature technologies to larger firms. Stuart and Podolny (1996) show that large firms tend to innovate along standard and well-explored fields.

Baumol (2002b) distinguishes the activity of innovation along established trajectories explicitly from the entrepreneurial function which consists rather in the search for new opportunities. While innovation processes by large firms are quantitatively more important since they imply large investments and they generate a larger stream of patents, a number of systematic studies have provided evidence that breakthroughs and new products are rather introduced by small and young firms, i.e. by entrepreneurs.[1] In that sense Baumol and Oates (1988) refers to innovation as an integrated process based on a division of labor between small firms, who launch new products and introduce new technologies, and large firms, who take on these ideas and develop them. Hence entrepreneurial firms and large firms coexist in what Baumol (2002a) calls a "*David-Goliath Symbiosis*". In that respect, entrepreneurship plays an important role for the economic dynamics and for the growth process of an economy.

The aim of this paper is to map this dichotomy into a model of endogenous growth, i.e. to model growth as an integrated process that is based on both types of innovation: variety expansion and quality improvement. We attribute the function of variety expansion to entrepreneurs while quality improvement is executed through the R&D activity of incumbent firms. The paper is organized as follows.

The following section discusses the role of entrepreneurship in the innovation process. Section 3 presents the model. Section 4 analyzes the implications of the model and highlights the role of entrepreneurs by comparing equilibrium dynamics with and without entrepreneurial activity. It is shown that sustainable growth does not require entrepreneurs but is greatly enhanced by it. Section 5 concludes.

2 Entrepreneurship, Opportunity Recognition and Economic Growth

At least since Walras (1874), entrepreneurship has been considered at the heart of opportunity recognition. Walras considered the function of the entrepreneur as seeking arbitrage opportunities. As such, the entrepreneur is the driving force behind the tâtonnement process that leads to the general equilibrium in the Walrasian model. Once the equilibrium is attained, however, the entrepreneur is no longer interesting and this is presumably why the usual analysis of equilibria seems independent of the entrepreneurial function. The Walrasian framework of general

[1] e.g. Scherer (1980) or CHI Research Inc. (2002). The U.S. Small Business Administration (1995, p.114) enumerates some 70 important innovations by small firms in the 20th century, ranging from low-tech innovations such as the Zipper or Bakelite to high-tech ones such as the Nuclear Magnetic Resonance Scanner or the Microprocessor.

equilibrium analysis describes the process in markets for existing products or technologies and is therefore independent of newness. Innovation or the introduction of new products or technologies would introduce (temporary) disequilibria into the Walrasian framework.

Schumpeter (1911, 1942) put the function of the entrepreneur central in the explanation of the dynamics capitalist economies. In Schumpeter the entrepreneur is the agent that "recognizes and realizes new economic opportunities", where opportunities were not only potential new products but also new production processes and opportunities in marketing and reorganization. Hence Schumpeter explicitly introduced *novelty* or new knowledge as the source of economic dynamics.

In more mainstream growth theory the importance of innovation and novelty was also realized when Solow (1957) showed the inability of more traditional production factors, i.e. capital and labor to account for economic growth. One of the main assumptions underlying modern endogenous growth theories is therefore that new knowledge behaves like a public good, i.e. it is non-exhaustive and non-excludable. This implies that the stock of existing knowledge and the newly created knowledge is available (i.e. spills over) automatically to all economic agents. In that respect, the properties of knowledge differ fundamentally from the "traditional" production factors. To explain the (sustained) generation of new knowledge, however, the role of the entrepreneur, as the agent commercializing such knowledge, was reduced to a mere mechanical one. All new knowledge simply is commercialized and causes innovation.

As Arrow (1962) already pointed out, however, new knowledge differs from the traditional production factors not only by its public goods characteristics, it is also inherently uncertain. By uncertainty, Arrow meant the fact that it is a priori unknown if newly generated knowledge can be transferred successfully into a viable innovation, be it a new product or any other innovation. Indeed, one can think of the stream of new knowledge arriving at a certain time period as involving different levels of uncertainty. For some of the new knowledge, its usefulness, hence the possibility of transforming it into a commercially viable new product is obvious to all involved in the production process. Think e.g. of quality improvements of existing products. On the other end of the "uncertainty scale" is new knowledge whose usefulness is not obvious at all, i.e. ideas about radically new products or technologies. Hence this knowledge is rather distant from existing products. Here, we can think of new knowledge that can be either very useful, indeed potentially revolutionizing[2], or useless and entirely inapplicable.

With increasing uncertainty, the *variance* of the economic benefits of new knowledge increases. Entrepreneurs are the agents who developed a vision about how to transfer new and uncertain economic knowledge into potentially viable products

[2]In the extreme case, these are technologies that are able to start up a new innovation life cycle in the sense of Gort and Klepper (1982).

and are bearing the risks that are involved with this process. By doing so, they explore new knowledge that otherwise would remain unexplored. They are therefore instrumental in the knowledge spill over and commercialization process in the economy. Moreover, unlike perhaps the new knowledge creators, they are primarily interested and entitled to the private benefits of knowledge commercialization. Consequently, entrepreneurship will foster economic performance.[3]

Summarizing this discussion, we state that the function of the entrepreneur is to seek arbitrage and innovation opportunities and to be prepared to pursue these opportunities and bear the risks involved in this enterprise. Although this importance has widely been acknowledged, the entrepreneur is absent in formal models of growth, even in models of radical innovation through variety expansion. As Baumol (1968) observed:

> *In recent years, while the facts have apparently underscored the significance of his* [the entrepreneur's] *role, he has at the same time virtually disappeared from the theoretical literature.* Baumol (1968, p. 64)

Certainly growth theory has made great progress since the publication of this quote, however we argue with Barreto (1990) that entrepreneurship still does not occupy the role that it deserves in growth theory. As seen from a different point of view (e.g. by Bianchi and Henrekson, 2005 and Acs and Sanders, 2007), entrepreneurship *is* present in models of the endogenous growth theory, but these models do not make a distinction between R&D and entrepreneurship in the innovation process, leading to biased policy implications.

In this paper, we develop a model that explicitly makes this distinction, hence considers two types of innovative activity: Entrepreneurship and R&D. While we define the function of entrepreneurship as the introduction of new products to the economy, the function of R&D is to improve existing products. With this approach we integrate two approaches to modeling innovation in endogenous growth models: Variety Expansion and Quality Improvement, relabeling variety expansion "entrepreneurship". Our model is not unique in combining different types of innovation. Helpman and Trajtenberg (1994) and Van Zon *et al.* (2003) suggest a model that is based on "drastic" versus "incremental" innovation in general purpose technologies. However, their motivation is to model cyclical behavior in the evolution of new technologies. Our purpose is to model the interplay between variety expansion and quality improvement, between exploration through entrepreneurship and exploitation through quality improvement and their respective impacts on growth.

Our entrepreneurs can be positioned clearly between public knowledge creation, that for simplicity is assumed to evolve gradually and autonomously, and product improvements, that are the domain of profit driven corporate R&D. The

[3] Audretsch, Keilbach and Lehmann, 2006, denote this process the *Knowledge Spillover Theory of Entrepreneurship*). They find repeated evidence that entrepreneurship is conducive to economic growth in interaction with the presence of R&D.

entrepreneurial function in our model is to combine ideas from the knowledge stock into opportunities and then bring new products to the market. Note that we consider entrepreneurship and R&D as *functions* rather than *persons*, i.e. we do not claim that entrepreneurs do not engage in R&D at one point in time or that established firms do not behave in an entrepreneurial way. On the basis of this model, however, we compute the equilibrium distribution between the two types of innovative activities: Entrepreneurship and R&D, and compare the (utility) growth rate with and without entrepreneurship to stress its importance.

3 The Model

We consider an economy in which consumers consume a range of goods and value both the quality and quantity consumed. These goods are produced by incumbent firms that set prices under monopolistic competition and hire workers to produce the final goods. Both the range of goods and the quality of existing goods can be augmented through entry and in house R&D respectively. The active population is therefore engaged in one of three activities:

$$1 = L + R + N \tag{1}$$

where 1 is the (normalized) population involved in the economic process, L is labor involved in production, R is employees involved in R&D and N is individuals acting as entrepreneurs. Production is simply generating output using known techniques and practices. R&D at any point in time is done by the existing firms to find improvements upon their own product design. Entrepreneurs are people that undertake the investments to set up a new venture and enter the market with a new variety. In the next sections we present each of these sectors and their maximization problem. then we analyze the equilibrium, the steady state and comparative static and transitional dynamic properties of the equilibrium.

3.1 Producing Sector

The incumbent firms use labor to produce n diversified and existing products. Each product i has a certain quality q_i assigned and comes with a corresponding price p_i and consumption level c_i. assuming that consumers care about both quality and quantity of a particular product type and are willing to substitute quantity for quality at the product type level, we can use a standard Dixit-Stiglitz love-of-variety instant utility function, augmented with variety specific quality parameter consumers solve

$$\max_{c_i} : \left(\int_0^n q_i^{1-\alpha} c_i^{\alpha} di \right)^{1/\alpha} \quad \text{s.t.} \quad \int_0^n c_i p_i di \leq E, \tag{2}$$

with $0 \leq \alpha \leq 1$. Here we have assumed that consumers substitute quality for quantity at elasticity 1 at the product type level and have a constant elasticity of

substitution across product types.[4] E is total expenditure on consumption. It can already be verified in the utility function that economic growth can come from 3 distinct sources. Variety expansion through entry, quality improvement on existing product types and regular increases in consumption volumes. To derive the instant global demand functions for all current and future goods in this CES-utility function is straightforward:[5]

$$c_i^D = q_i \left(\frac{p_i}{P}\right)^{\frac{1}{\alpha-1}} \frac{E}{P} \quad \text{where} \quad P \equiv \left(\int_0^n p_i^{\frac{\alpha}{\alpha-1}} q_i di\right)^{\frac{\alpha-1}{\alpha}}, \tag{3}$$

where P is a quality adjusted exponentially weighted price index that can be defined a the minimum cost of one util. Production takes place under monopolistic competition such that producers can set prices to maximize profits. At every point in time they take demand and the quality of their product as given. Hence producers solve:

$$\max : \pi_i = c_i \cdot p_i - w \cdot l_i \tag{4a}$$

$$s.t. : c_i = c_i^D \tag{4b}$$

$$s.t. : y_i = bl_i, \tag{4c}$$

π_i being profits of firm i, w is the wage level and l_i is the labor force employed by i to produce c_i. (4b) makes sure that the market clears and (4c) is a simple linear production function with labor as single input. This condition excludes the possibility for steady state growth from increases in production volumes as the productivity parameter b is given and the level of employment fixed by the absence of population growth in the model.[6] Solving the set of equations (4) yields the equilibrium price of product i:

$$p_i = \frac{w}{\alpha b} \tag{5}$$

and the equilibrium profit of producing it

$$\pi_i = \frac{(1-\alpha)Eq_i}{nQ} \quad \text{where} \quad Q \equiv \frac{1}{n}\int_0^n q_i di. \tag{6}$$

Equation (6) makes clear that the profit of product i,π_i, increases with its quality q_i. Positive profits will also create an incentive to enter the market, i.e. to introduce a new product with a given initial quality and with unknown demand. We denote agents that do so as entrepreneurs. Note that entrepreneurs are defined as such only for the time it takes them to set up a new production facility and start producing and selling the new variety. After having done so successfully, the entrepreneur becomes an incumbent producer (or sells the firm to start a new venture).

[4] Also note that, although in principle it would be possible to consume different quality levels of the same product type, production of each type by a monopolistic firm implies only one quality level (the highest) will be available in equilibrium.

[5] See for example Grossman and Helpman (1991b).

[6] See Acs and Sanders (2007) for a model that allows for productivity (but not quality) improvements

3.2 Knowledge, R&D and Entrepreneurship

In the previous section it was shown that quality improvements to existing product types and the introduction of entirely new types are both profitable activities. However, in contrast to more traditional quality ladder models, such as Aghion and Howitt (1992) quality improvement on existing products in our model are *only* valuable to the incumbent producer. As incumbents hold the patent protecting the original design, command the resources for production, developed the sales and distribution channels etc. etc. we assume that quality improvements on existing product types are particularly valuable to the incumbents and hence they have an incentive to invest in R&D to develop such improvements. To do so they hire R&D workers who use common and firm specific knowledge to generate the next quality improvement.

For now let us assume that the stock of common knowledge is K. As we are not primarily interested in the sources of growth but rather want to focus on the role of the entrepreneur in economic growth, we assume this rate to be exogenously given by g.[7]

Then assume that knowledge positively affects the increase of quality of existing products in the R&D process. We specify this activity as

$$\dot{q}_i = hK^{\gamma}R_i^{\gamma} \tag{7}$$

where $0 \leq \gamma \leq 1$. K is the existing body of knowledge in the economy that augments R_i, the level of R&D effort in firm i. The marginal productivity of effective R&D is decreasing in the level of R&D (expressed by R_i) itself. Note also that the rate of quality improvement decreases in the level of quality achieved at the firm level. Quality improvement is thereby effectively excluded as a source of steady state growth in this equation.[8] Equation (7) thus deviates from standard quality ladder models, where a given level of effort yields a constant *rate* of quality improvement due to such intertemporal spillovers of knowledge, but this assumption can be justified as improving quality in already high quality products is typically harder than thinking up quality improvements on low quality products.[9]Instead we have now modeled R&D as receiving a spillover form outside knowledge creation (which is exogenous) and producing firm specific output in the form of quality that is a stock variable. The incumbent firm is not engaged in a patent race or tournament, is not taking any risks by doing R&D and is not subject to displacement and rent stealing as in Aghion and Howitt (1992).

[7] In another version of the model, we endogenized g as a positive function of R in a specification à la Romer (1990). The outcome of the model is not affected by this specification. We therefore keep g exogenous for tractability.

[8] The rate of exogenous knowledge growth in steady state equilibrium will be exactly offset by the spreading of a given number of R&D workers over a growing number of firms, which in steady state will also expand at rate g (See below). The result is thus a constant increment in quality that vanishes in relative terms.

[9] We have, for that reason, not included q_i on the right hand side of equation (7).

The second role of knowledge in our model is to determine the number of potential products n^P. Consider n^P as the number of *opportunities* that can be developed out of the current state of knowledge K. Hence:

$$n^P = \xi K \tag{8}$$

Opportunities include unrealized as well as realized products, i.e. $n \subset n^P$. However, as long as $n < n^p$, there exist unexploited opportunities hence room for entrepreneurial activity. By the act of starting a new venture, an entrepreneur introduces a new product in the market. Hence he is developing a previously unrealized idea out the pool of potential products n^P. Formally this activity can be represented by:

$$\dot{n} = a(n^P - n)N^\beta \tag{9}$$

where $0 \leq \beta \leq 1$. N is the level of entrepreneurial effort. Note that this introduces strong diminishing returns with respect to n as the marginal productivity of the entrepreneur falls to 0 when all opportunities are exploited, i.e. if $n^P - n \rightarrow 0$. This puts an upper limit on n that by Equation (8) expands at rate g. Equation (9) therefore implies that variety expansion in the model is restricted in the long run to the rate at which knowledge expands.

By the assumed symmetry in utility and the fact that profits are made by incumbent firms, there is an incentive for entrepreneurs to enter the market, i.e. to start up a new venture. This implies that a new variety is introduced to the market i.e. n increases. The value of realizing one commercial opportunity is given by the expected discounted profit flow that the product i yields at some initial quality level q_{i0}, which can be written as[10]:

$$v_n(t) = \int_t^\infty e^{-r(\tau - t)} \pi_i(q_{i0}, \tau) d\tau. \tag{10}$$

q_{i0} will be realized only when the product is first introduced and is therefore ex ante unknown to the entrepreneur. From equation(6) we see that increasing variety, n, and average quality, Q, erode the profits of firms. Moreover, it is unknown ex ante how fast competitors will improve the relative quality of their products, how fast variety expands and how fast consumption expenditure grows. Hence the rate of profit erosion is unknown.

For now we abstract from any uncertainty that is inherent to the introduction of new products to the market to illustrate the essential mechanisms in the model. Hence we assume that q_{i0} is a known parameter and entrepreneurs form rational expectations, which implies that they have on average correct expectations on future profit erosion rates. As we will show below, they are constant in steady state

[10] as all firms are symmetric they will invest along the same optimal R&D investment path and increase their level of quality at the same rate. Due to the diminishing returns to R&D labor they earn additional inframarginal profits that are, however, proportional to entry-level profits. It would be more precise but also much more complicated to evaluate the value of entry at the average expected quality level.

equilibrium and hence rational expectations imply that entrepreneurs expect constant profit erosion rates. In that case we show in appendix A on page 16 that the marginal value of a business opportunity (10) can be rewritten to:

$$v_n(q_{i0},t) = \frac{\pi_i(q_{i0},t)}{r - \dot{E}/E + \dot{Q}/Q + \dot{n}/n} = \frac{(1-\alpha)EQ^{-1}q_{i0}n^{-1}}{r - \dot{E}/E + \dot{Q}/Q + \dot{n}/n}. \tag{11}$$

Equation (11) states that the instant profit flow π_i is discounted against the interest rate plus the rate of average quality improvement plus the introduction rate of new products by entrepreneurs, which by (6) is equal to the rate of profit erosion due to variety expansion and quality improvements in substitutes.

The value firm i derives from adding to the quality index of product i at the margin is given by the derivative of (11)with respect to q_i.[11] As the effect of one product's quality index on Q is negligible we obtain:

$$v_q(q_i,t) = \frac{1}{r - \dot{E}/E + \dot{Q}/Q + \dot{n}/n}\frac{d\pi_i(q_i,t)}{dq_i} = \frac{(1-\alpha)EQ^{-1}n^{-1}}{r - \dot{E}/E + \dot{Q}/Q + \dot{n}/n}. \tag{12}$$

This value is what motivates incumbent firms to hire R&D workers and have them generate quality improvements following the process in Equation (7).

3.3 Equilibrium

The equilibrium in the model requires the market clearing conditions for labor, entrepreneurship and research. If we assume that the opportunity costs for entrepreneurs and R&D workers are given by the general wage level, we can calibrate the productivity parameters a, b and h to obtain a reasonable allocation of labor over the various activities in the economy. Of course that implies that entrepreneurs, R&D workers and production workers are perfect substitutes, which we certainly to not wish to claim. Still, as long as we assume that the wage in production provides the opportunity costs to R&D workers and entrepreneurs and there is free entry in both occupations, the result is similar as the production wage puts a floor in the marginal revenue of engaging in entrepreneurial activity and doing research. Therefore this assumption is the best we can do in a model with homogeneous labor.

Having made that point, the labor market clears when equation (1) is fulfilled. The demand for labor can then be derived from inverting production function (4c), substituting for quantities using demand in (3) and prices in (5) yields:

$$l_i = \frac{1}{n}\frac{\alpha E}{w}\frac{q_i}{Q}. \tag{13}$$

Integrating over all n yields the aggregate labor demand for production:

$$L = \frac{\alpha E}{w} \tag{14}$$

[11] This is the case because the incumbent is the only one to do such R&D by assumption.

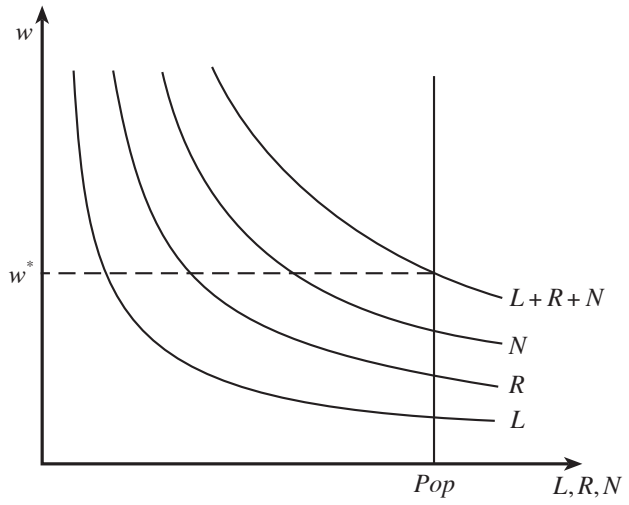


Figure 1: Equilibrium wage level

To obtain the demand for entrepreneurship and R&D, the marginal value product value of these activities is set equal to the wage level w. Assuming all entrepreneurs expect to enter the market at the same quality level and rearranging yields:

$$N = \left(\frac{w}{a\beta}\right)^{\frac{1}{\beta-1}} v_n(q_{i0}, t)^{\frac{1}{1-\beta}} \left(\xi K - n\right)^{\frac{1}{1-\beta}} \tag{15}$$

It is worth noting that entrepreneurial activity is negatively related to the wage in production and positively related the the marginal value of a business opportunity v_n. From (11) we know that the equilibrium level of N increases with the profit level and with the growth rate of E but decreases with the growth rate of Q and n. In addition N responds positively to increases in the knowledge stock. The intuition is that more knowledge makes entrepreneurs more likely to succeed. Similarly, the demand for R&D employees we find for incumbent firm i:

$$R_i = \left(\frac{w}{h\gamma}\right)^{\frac{1}{\gamma-1}} v_q(q_i, t)^{\frac{1}{1-\gamma}} K^{\frac{\gamma}{1-\gamma}}$$

which, integrated over n yields:

$$R = \left(\frac{w}{h\gamma}\right)^{\frac{1}{\gamma-1}} v_q(q_i, t)^{\frac{1}{1-\gamma}} nK^{\frac{\gamma}{1-\gamma}}. \tag{16}$$

Hence the level of R&D activity is also negative in the general wage level and positive in the knowledge stock. Also it responds positively to higher expenditure growth and negatively to increases in the average quality level. The level of R&D activity, however, we know from (12) that R does not does not respond to the level of profits but to the marginal increase in profit that quality improvement allows. In addition, the number of existing varieties now has a positive impact as more varieties imply more varieties that need quality enhancing R&D.

As (14), (15) and (16) are all decreasing in the wage, there is a unique wage level that clears the labor market. This can be illustrated in a figure such as Figure 1. Due to the different elasticities of the curves, however, it is not possible to compute the analytical solution for the equilibrium wage. To prove the existence of a steady state equilibrium, however, this is not required. We can infer its existence from assuming its properties and proving that these assumption yields a stable equilibrium in the labor market.

3.4 Steady State

For the steady state to be stable, the allocation of the active population to the aggregates L, R and N must be stable.[12] Hence this implies that:

$$\frac{\dot{L}}{L} = \frac{\dot{R}}{R} = \frac{\dot{N}}{N} = 0. \tag{17}$$

Given that the population does not grow, the level of production, research and entrepreneurship have to be constant in the steady state. Taking time derivatives and computing the growth rate of (14), (15) and (16) yields the following conditions for the steady state:

$$\frac{\dot{E}}{E} = \frac{\dot{w}}{w} \tag{18a}$$

$$\frac{\xi\dot{K} - \dot{n}}{\xi K - n} = \frac{\dot{Q}}{Q} + \frac{\dot{n}}{n} \tag{18b}$$

$$\gamma\left(\frac{\dot{K}}{K} - \frac{\dot{n}}{n}\right) = \frac{\dot{Q}}{Q} \tag{18c}$$

Using the fact that in any steady state the growth rate of n (the number of varieties) must be equal to the growth rate of n^P (the number of opportunities) and therefore equal to the growth rate of K (the knowledge stock), implying that the difference between K and n grows at the same rate g. Combining this with the set of equations (18) yields the results that a steady state may exist as long as

$$\frac{\dot{w}}{w} = \frac{\dot{E}}{E} = r - \rho; \tag{19}$$

and

$$\frac{\dot{K}}{K} = \frac{\dot{n}^P}{n^P} = \frac{\dot{n}}{n} = g \quad \text{and} \quad \frac{\dot{Q}}{Q} = 0. \tag{20}$$

Normalizing the expenditure on consumption $E = 1$ yields the growth rate of utility in the economy

$$g_U = -\frac{\dot{P}}{P} = \frac{1-\alpha}{\alpha} g. \tag{21}$$

[12] In addition we use the condition that the interest rate is constant in the steady state, which can be derived from inter temporal utility maximization, using a standard time separable CIES-utility function.

Given conditions (20) for the steady state, we derive from equation (9) that the total level of entrepreneurial activity in the steady state, N^{SS}, must be equal to:

$$N^{SS} = \left(\frac{g}{a} \frac{n}{n^P - n} \right)^{1/\beta}, \tag{22}$$

which states that N^{SS} increases in g, the growth rate of knowledge, and therefore the growth rate of potential products (from equations 20). On the other hand, the level of the steady state of N increases as n approaches n^P. The interpretation is that as the number of new varieties in the model approaches its maximum value, the level of entrepreneurship has to increase to maintain the equilibrium rate of variety expansion constant at g (from 9).

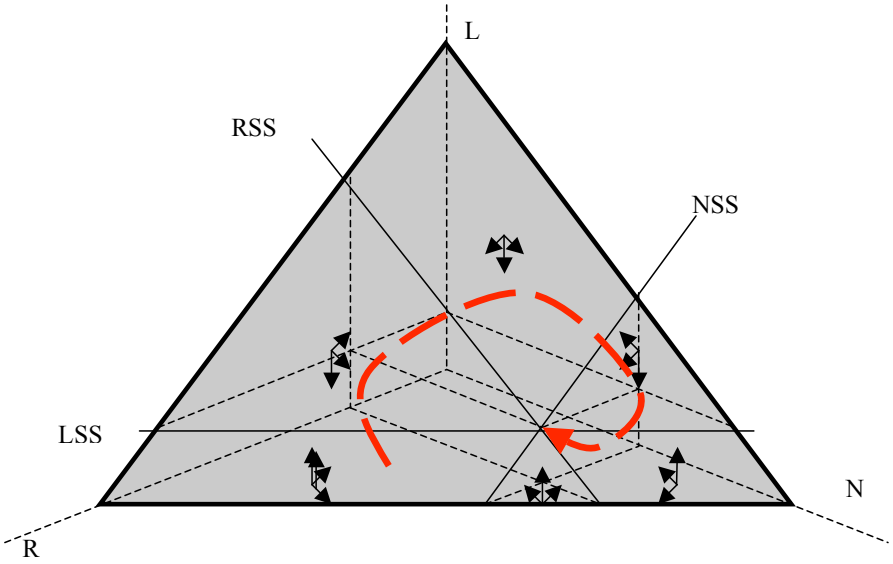


Figure 2: Dynamics and Steady State Distribution in the Labor Market on the Unit Simplex

In a similar way, we derive from equation (7) the steady state level of R as:

$$R^{SS} = \frac{n}{K} \left(\frac{gQ}{h} \right)^{1/\gamma} \tag{23}$$

The steady state demand for labor, L^{SS} can then be derived from (1). Figure 2 gives a graphic representation of the steady state distribution.

3.5 Dynamic Properties of the Steady State

To analyze the dynamic properties of the model, we derive the equations of motion for Q, n and K. The formal derivation of these equations is given in Appendix C. There we show, that the system will converge to a stable, non trivial equilibrium in the $(Q, n/K)$ space, i.e. a point with positive long term growth rate of utility and

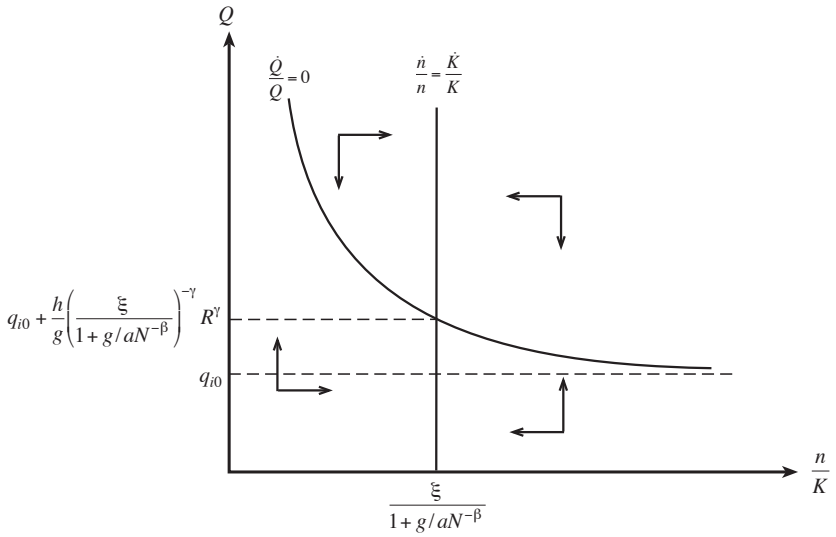


Figure 3: Phase Space of the Dynamics in the Model

with a positive average quality of the products in the economy. Figure 3 depicts this dynamic equilibrium.

In the steady state the allocation of labor to production, R&D and Entrepreneurship is stable but not optimal. Without computing the exact proofs we can see from equation 9 that more varieties will ceteris paribus reduce the productivity of entrepreneurial labor. This "fishing in the pond"-effect is a negative externality in our model and it implies that in decentralized equilibrium there is more than optimal entrepreneurship. By the same token there are sub-optimal levels of production and R&D. This result is counterintuitive and follows from the simplifying assumptions on knowledge accumulation. If we had assumed positive knowledge spillovers from entrepreneurial activity and R&D into the public knowledge stock, the results could easily be reversed. See for example Acs and Sanders (2007) for a model that has such explicit knowledge spillovers. As the aim of this paper is to analyze the contribution of entrepreneurship and its interaction with R&D we need not consider optimality.

4 Assessing the Contribution of Entrepreneurship

To asses the contribution of Entrepreneurship to growth in our model, we set $q_{i0} = 0$ such that in equilibrium $N = 0$ and therefore (from equation 9) $\dot{n} = 0$, hence no new product variety will be introduced in the economy. However(17) still defines the steady state of the model. Also the demand for R&D is still given by (16). Given that now $\dot{n}/n = 0$, taking time derivative of (16) and requiring them to equal 0 yields the following conditions for the steady state:

$$\frac{\dot{E}}{E} = \frac{\dot{w}}{w} \tag{24a}$$

$$\gamma\left(\frac{\dot{K}}{K}\right) = \frac{\dot{Q}}{Q} \tag{24b}$$

Solving for the steady state of the level of R&D, we obtain:

$$R^{SS} = \frac{n}{K}\left(\frac{\gamma g Q}{h}\right)^{1/\gamma} \tag{25}$$

which is a constant since K is assumed to grow at g and therefore (by 24b), Q grows at $\gamma \cdot g$.[13] Again normalizing $E = 1$ we obtain

$$g'_U = \frac{1-\alpha}{\alpha}\gamma g, \tag{26}$$

which is smaller than (21) by a factor γ, the output elasticity of the R&D industry. Hence the contribution of Entrepreneurship to economic growth is

$$(1-\gamma)\frac{1-\alpha}{\alpha}g,$$

which is always positive for $g > 0$ and $\gamma < 1$. Note that with $N = 0$, the mechanism of growth in the model has shifted from variety expanding growth to purely quality enhancing growth, however both are ultimately driven by (exogenous) knowledge accumulation.

5 Summary and Conclusions

While it is readily acknowledged in the literature that entrepreneurship plays an important role in the process of economic growth, existing growth models do not explicitly consider this the entrepreneur's function. In this paper, we model economic growth as a function of two distinct innovation processes, variety expansion, i.e. the introduction of new products through entry, and quality enhancement of existing products. While the latter function is ascribed to R&D efforts that are managed within of existing firms, the first function is executed by entrepreneurs.

From a model building point of view, we show that it is possible to integrate both types of innovation process into a growth model. The model degenerates into a standard endogenous growth model with quality enhancement if entrepreneurs so not exist. On the other hand, the model degenerates to a standard model with variety expansion if the R&D sector does not exist. Therefore, our model can be considered as a *generalized innovation driven growth model.* To our knowledge, this

[13] Note that this implies that the growth of the model *without* entrepreneurship is entirely driven by quality improvement other than in the model *with* entrepreneurship, where growth is driven by variety expansion and where quality improvement converges to 0 (equation 20). Hence the corner solution of the model where $N = 0$ yields a qualitatively different outcome.

has not been done previously although some efforts have been made to model different types of innovation in one model.[14]

Based on this model, we can show that the economy converges to a stable non-trivial distribution among the two types of innovative activity: R&D employment and entrepreneurship. We can also show, that this distribution leads to a stable non-trivial path of steady state growth of utility in the economy. We finally show formally that entrepreneurship does make a positive contribution to the process of growth of utility.

Based on these findings, it can be argued that scarcity of entrepreneurial talent and/or adequately trained R&D workers will slow down an economy. If, for some reason, entrepreneurial activity falls short of its steady state level, the level of R&D activity will actually be too high as the rate of variety expansion is below its steady state level and this increases the value of quality improvement. On the other hand, a lack of R&D capacity will cause a lower rate of aggregate quality improvement, making entrepreneurial activity artificially attractive.

As the US and Europe can both access the same pool of knowledge, respective relative shortages of R&D capacity and entrepreneurial spirit may explain the apparent specialization in entrepreneurial and corporate innovation respectively. It can also be verified in the model that increasing ξ, i.e. improving the permeability of the knowledge filter generates a one time increase in growth and raises the economic to a higher level of utility but not to a permanent increase in the growth rate.[15]

[14] see for example Van Zon *et al.* (2003) for a model that combine GPT and incremental innovations and Sanders (2005) for a model that combines product and process innovations

[15] Acs, Audretsch, Braunerhjelm and Carlsson (2003)

A From Equation (10) to Equation (11)

Entrepreneurs, when introducing a new product to the market, face uncertainty. In our model we have assumed symmetry among products in the utility function, which implies there is always a positive, actually infinite, demand for new varieties. Of course this is a simplification and uncertainty runs deeper than even our model allows. Yet a lot of uncertainty can still be introduced if required. We abstract from doing so to work out the fundamental properties of the model, but in no way want to claim that our formalization of the entrepreneurial act captures this fundamental aspect of it. The reward for a successful entrepreneur is the flow of rents, monopoly profits, that he can earn by bringing his new product variety to the market. Discounted to the present this flow of profits is given by the expression:

$$v_{n+1}(t) = \int_0^\infty e^{-r(\tau-t)} \pi_{n+1}(q_{n+1}(\tau), \tau) d\tau$$

A firm that enters can earn positive profits at the initial quality level but may increase that profit flow even further by investing in R&D. As that option requires entry and increases the discounted future profit flow when R&D faces diminishing returns to labor, the expected rewards to the entrepreneur are higher than the discounted value of an infinite profit flow at the initial quality level. we assume for simplicity, however, that this downstream profit flow is so uncertain that the entrepreneur does not take it into consideration when deciding to enter. This and the assumption that firms enter with a known and given initial quality level for all new goods, allows us to write the integral as in equation (**??**):

$$v_n(t) = \int_0^\infty e^{-r(\tau-t)} \pi_i(q_{i0}, \tau) d\tau$$

Recall from (6) that profits, without further quality improving investments, are given by:

$$\pi_i(t) = \frac{(1-\alpha)E(t)q_i(t)}{n(t)Q(t)} = \frac{(1-\alpha)E(t)q_{i0}}{n(t)Q(t)}$$

Quality improvements are costly in our model so once in operation the decision to invest in them is a new and separate decision. The (discounted) additional profits of such investments will, in equilibrium, just offset the costs and therefore have no impact on the decision to bring the product to the market. Having said that, it is clear that the growth rate of profits equals:

$$\frac{\dot{\pi}_i(t)}{\pi_i(t)} = \frac{\dot{E}_i(t)}{E_i(t)} - \frac{\dot{n}_i(t)}{n_i(t)} - \frac{\dot{Q}_i(t)}{Q_i(t)}$$

Which entrepreneurs with rational expectations know will be constant in the steady state. But is the growth rate is expected to be constant, profits at time t are given by:

$$\pi_i(q_{i0}, t) = e^{\left(\frac{\dot{E}_i(t)}{E_i(t)} - \frac{\dot{n}_i(t)}{n_i(t)} - \frac{\dot{Q}_i(t)}{Q_i(t)}\right)(t-t_0)} \pi_i(q_{i0}, t_0)$$

Dropping the time arguments on constant growth rates allows us to write the integral as:

$$v_n(t) = \int_0^\infty e^{-r(\tau-t)} e^{\left(\frac{\dot{E}}{E} - \frac{\dot{n}}{n} - \frac{\dot{Q}}{Q}\right)(\tau-t)} \pi_i(q_{i0}, \tau) d\tau$$

Which solves easily into (11) Q.E.D.

Similar reasoning applies to (12), where the value of increasing quality is equal to the discounted marginal profit from higher quality

B Stability of the Steady State Allocation of Labor

The stability of the steady state can be shown by deriving the sign of the impact on entrepreneurial activity N and research and development R of rising n and Q out of steady state equilibrium. The intuition is straightforward. If N exceeds its steady state level, the growth rate in n, by equation (10) is also higher than its steady state value. This implies that the economy will return to steady state only if a rise in n reduces the deviation from equilibrium entrepreneurial activity. Formally we check:

$$\frac{d(N(t) - N^{SS})}{dn(t)} < 0.$$

Similarly for R&D:

$$\frac{d(R(t) - R^{SS})}{dQ(t)} < 0.$$

If these conditions hold we know that production labor is also adjusting in the right direction and the steady state is stable. Substituting for $N(t)$ using equations (15) and (11) and for N^{SS} using (22) we find:

$$N(t) - N^{SS} = \left(\frac{w}{a\beta}\right)^{\frac{1}{\beta-1}} (\xi K - n)^{\frac{1}{1-\beta}} \left(\frac{(1-\alpha)Eq_{i0}}{nQ}\right)^{\frac{1}{1-\beta}} \left(r - \frac{\dot{E}}{E} + \frac{\dot{n}}{n} + \frac{\dot{Q}}{Q}\right)^{\frac{1}{\beta-1}} - \left(\frac{R^\phi}{a} \frac{n}{\xi K - n}\right)^{\frac{1}{\beta}}.$$

The derivative with respect to n is given by:

$$\frac{d(N(t) - N^{SS})}{dn} = \frac{\left(\frac{aEq_{i0}(1-\alpha)\beta(\xi K - n)}{n^2 Q\left(r - \frac{\dot{E}}{E} + \frac{\dot{n}}{n} + \frac{\dot{Q}}{Q}\right)w}\right)^{\frac{1}{1-\beta}} (n - 2\xi K)\beta + \left(\frac{R^\phi n}{a(\xi K - n)}\right)^{\frac{1}{\beta}} (\beta - 1) K\xi}{n(1-\beta)\beta(\xi K - n)},$$

of which the denominator is larger than 0, as well as the large terms between brackets in the numerator. It is then easily verified that the derivative is negative and hence the number of entrepreneurs will return to the steady state level when the economy finds itself out of equilibrium.

Similarly we can present the derivative of R&D employment with respect to average quality levels:

$$\frac{d(R(t) - R^{SS})}{dQ} = -\frac{n}{Q}\left(\left(\frac{R^\phi Q}{h}\right)^{\frac{1}{\gamma}} K\gamma + \frac{1}{1-\gamma}\left(\frac{EhK^\gamma(1-\alpha)\gamma}{n^2 Q\left(r - \frac{\dot{E}}{E} + \frac{\dot{n}}{n} + \frac{\dot{Q}}{Q}\right)w}\right)^{\frac{1}{1-\gamma}}\right)$$

Again it is easily verified that this expression is smaller than 0, which establishes the stability of the steady state as illustrated by the arrows in Figure 1.

C Dynamic Properties of the Model

Equation (6) states the *index of average quality* of all products in the economy

$$Q(t) \equiv \frac{1}{n(t)} \int_0^{n(t)} q_i(t)di.$$

Deriving this index with respect to t yields

$$\begin{aligned} \frac{dQ(t)}{dt} &= -\frac{\dot{n}(t)}{n(t)}Q(t) + \frac{\int_0^{n(t)} \dot{q}_i(t)di + q_{i0}\dot{n}(t)}{n(t)} \\ &= (q_{i0} - Q)\frac{\dot{n}}{n} + \bar{\dot{q}}_i = 0 \text{ for } \dot{q}_i = \bar{\dot{q}}_i \forall i = 1, 2, \ldots, n \end{aligned}$$

Substituting in the R&D quality improvement function (7) we have:

$$h\left(\frac{n}{K}\right)^{-\gamma} R^{\gamma} = (Q - q_{i0})\frac{\dot{n}}{n}$$

Solving for Q and substituting g for the growth rate of n yields:

$$Q = q_{i0} + R^{\gamma}\frac{h}{g}\left(\frac{n}{K}\right)^{-\gamma}.$$

It can be verified that $Q(t)$ will increase over time if $Q(t)$ lies below this line: Let

$$\begin{aligned} \dot{Q}(t) &> 0 \\ (q_{i0} - Q(t))\frac{\dot{n}(t)}{n(t)} + h\left(\frac{n(t)}{K(t)}\right)^{-\gamma} R^{\gamma} &> 0 \\ h\left(\frac{n(t)}{K(t)}\right)^{-\gamma} R^{\gamma} &> (Q(t) - q_{i0})\frac{\dot{n}(t)}{n(t)} \\ \frac{h}{g}\left(\frac{n(t)}{K(t)}\right)^{-\gamma} R^{\gamma} + q_{i0} &> Q(t) \end{aligned}$$

Now consider the condition for n/K to be stable. Required is:

$$\dot{n}/n = \dot{K}/K = g$$

By the entrepreneurial production function (9) we know that:

$$\frac{\dot{n}(t)}{n(t)} = \frac{a(n^P(t) - n(t))N^{\beta}}{n(t)} = \frac{a(\xi K(t) - n(t))N^{\beta}}{n(t)} = \frac{a\xi K(t)N^{\beta}}{n(t)} - \frac{aN^{\beta}}{n(t)} = g$$

Solving for n/K yields:

$$\frac{n(t)}{K(t)} = \frac{\xi}{1 + g/aN^{-\beta}}$$

If n/K exceeds this value, n will grow at a rate below g implying n/K will fall:

$$\begin{aligned} \frac{\dot{n}}{n} &< g \\ \frac{a(\xi K - n)}{n}N^{\beta} &< g \\ \frac{\xi K}{n} - 1 &< g/aN^{-\beta} \\ \frac{\xi K}{n} &< g/aN^{-\beta} + 1 \\ \frac{n}{K} &> \frac{\xi}{g/aN^{-\beta} + 1} \end{aligned}$$

This implies the equilibrium in the graph depicted by Figure 3 is a stable attractor in the system. Arrows indicate the direction in which the system will move.

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