Inspecting the Poverty-Trap Mechanism
A Quantile Regression Approach

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Jens J. Krüger

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Friedrich Schiller University Jena
Carl-Zeiss-Str. 3
D-07743 Jena
www.uni-jena.de

Max Planck Institute of Economics
Kahlaische Str. 10
D-07745 Jena
www.econ.mpg.de

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Jens J. Krüger

Friedrich-Schiller-University Jena
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Department of Economics, Carl-Zeiss-Strasse 3, D-07743 Jena, Germany,
Tel.: +49 3641 943 203, Fax: +49 3641 943 202, E-Mail: jens.krueger@wiwi.uni-jena.de

Abstract:

The issue of poverty traps is assessed using quantile regression. For that an augmentation of the usual convergence regressions by quadratic and cubic terms is used with emphasis on curve fitting rather than parameter estimation. The results show that the generic mechanism leading to poverty traps predominantly applies to countries with relatively low levels of income per capita or per worker that simultaneously have low growth rates around and below the lowest quintile of the growth rate distribution. The validity of the results is supported by a nonparametric variant of quantile regression.

JEL classification: O1, O41, C14, C62

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1 Introduction

In their recent account of the empirical growth literature, Durlauf et al. (2005) again document the great diversity of international growth rates. Many less developed countries persistently have low growth rates whereas some experience miraculous high growth rates over several decades, in-between are the developed countries with sustained medium-level growth rates. This diversity is related to two important findings in empirical growth research. First, the absence of convergence in the sense of absolute β-convergence (Barro and Sala-i-Martin 1991, 1992) is a robust finding stating that relatively poor countries not systematically tend to grow faster than relatively rich countries. Second, the growth rate differences also bear the tendency towards clubs of countries that cluster together at different levels of per capita income, thereby generating the bimodal (twin-peaked) shape of the world income distribution (Quah 1996, 1997).

In this paper we reconsider the topic by taking a different look at the relation of per capita income growth and levels of per capita income. Given the divergence of per capita incomes and given the development of a bimodal world income distribution, a specific pattern of growth dynamics should be at work leading to multiple equilibria with a stable fixed point at low levels of per capita income. In that context the stable fixed point at low income is generally called a poverty trap. Those situations can be readily visualized by phase diagrams. Matsuyama (2008) is brief survey of the poverty-trap literature featuring the role of phase diagrams which are also the basis of the approach followed in the present paper.

The strategy of the present paper is to take a direct approach to assess the growth dynamics associated with poverty traps in an empirical representation of such phase diagrams. The analysis is in a curve-fitting spirit for investigating the scatter of growth rates and (log) income levels rather than estimating and interpreting specific regression coefficients. Both parametric and nonparametric regression methods and both in the form of usual mean regression as well as in the more general form of quantile regression are applied. The results imply that being trapped in a low-income equilibrium state is only a danger for countries with both a low level and a low growth rate of income per capita (or per worker).
Related analyses can be found in Fiaschi and Lavezzi (2003, 2007), Liu and Stengos (1999) and Kalaitzidakis et al. (2001). They share the common feature with the present paper that semi- or nonparametric regression methods are used to uncover nonlinearities in the growth process that may lead to poverty traps. The added value of the present paper is the application of quantile regression which allows to uncover different relations of income and growth at different quantiles of the growth rate distribution. This combined analysis of growth rates and income levels is in spirit of the state-space definition of Fiaschi and Lavezzi (2003, 2007), albeit implemented here in a rather different fashion.

The paper proceeds as follows: Section 2 briefly explains the basic mechanism leading to poverty traps and its representation in phase diagrams. This is followed by direct parametric and nonparametric estimates in section 3 and quantile regression estimates in section 4. Section 5 concludes.

2 The Basic Mechanism

From the various ways mentioned in Azariadis and Stachurski (2005) and Matsuyama (2008) of how a poverty trap may arise we want to focus here on one basic mechanism involved and confront this basic mechanism with the data. A simple possibility to generate poverty-trap dynamics is to state the law of motion of the capital intensity \( k \) as in Azariadis and Stachurski (2005)

\[
k_{t+1} = s \cdot A(k_t)k_t^\alpha \varepsilon_{t+1} + (1 - \delta)k_t,
\]

where \( s \) denotes the constant savings rate, \( \delta \) the depreciation rate and \( \varepsilon \) is a productivity shock. The function \( A(k_t) \) relates capital intensity and the level of productivity. Depending on the particular shape of this relationship a variety of dynamic developments are possible. This function can be interpreted as representing an externality associated with capital accumulation, for example.\(^1\) Basically, it has to be strictly positive, nondecreasing and

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\(^1\) See in particular Azariadis and Drazen (1990), Galor and Weil (2000) and Zilibotti (1995) for more elaborate models with multiple equilibria arising from the accumulation of either physical or human capital. Azariadis and Stachurski (2005) provide a comprehensive survey of the whole literature on poverty traps and a discussion of the various mechanisms leading to this phenomenon.
bounded (Azariadis and Stachurski 2004, assumption 3.2). Different returns-to-scale regimes in different ranges of capital intensity lead to capital intensity dynamics that are associated with multiple equilibria. In the formula it holds that per capita income $y_t$ is

$$y_t = A(k_t)k_t^\theta e_{t+1}.$$  

The dynamics of the capital intensity carry over to the dynamics of income per capita and its logarithm. Azariadis and Stachurski (2004) guide through an example. The dynamics of per capita income can be expressed in general form as

$$\Delta \ln y_{t+1} = \ln y_{t+1} - \ln y_t = T(\ln y_t, e_{t+1}).$$

This is also the setting in which Matsuyama (2008) discusses the subject of poverty traps. Two particular shapes of this relation (abstracting from the productivity shocks) that lead to the poverty-trap phenomenon can be illustrated by the phase diagrams shown in figure 1. Note that in contrast to the setting in Matsuyama with $\ln y_{t+1}$ on the ordinate and $\ln y_t$ on the abscissa (where the intersections of the curve describing the per capita income dynamics with the 45° line are defining the fixed points), here the intersections with the abscissa are relevant for the definition of a fixed point.

**Figure 1**
Phase Diagrams Associated with Poverty Traps
The left diagram shows the situation when the curve governing the dynamics has a wave-shape (variant 1). Three equilibria (A, B, C) appear with A and C stable and B unstable. An economy with a (log) per capita income lower than A will grow until it reaches the level A and will remain at that point. If the economy becomes subject to a positive technology shock that increases per capita income to some level between A and B it will inevitably be driven back by the dynamics to the stable point A. This is the poverty trap. Only if the economy is subject to a large technology shock (or a series of smaller shocks) that boosts per capita income beyond the level B it will be able to escape the poverty trap and to converge to the second stable equilibrium at point C.

Another possibility leading to a poverty trap is shown in the right diagram of the figure (variant 2). Here, the curve is u-shaped and an economy with a per capita income lower than A will again grow until it reaches the level A and will remain there. For any per capita income level between A and B the dynamics lead back to A since B is here also an unstable fixed point. Again, technology shocks large enough such that per capita income increases beyond B are required to escape the poverty trap. In contrast to the situation in the left diagram, there will be a sustained increase of per capita income once the level B is passed. This outcome would be in line with the literature on the transition from stagnation to growth, where a regime change occurs once a critical threshold is passed. See Galor (2005) as well as Galor and Weil (2000) for more on this kind of models.

Last, it should be mentioned that the way to assess absolute $\beta$-convergence (Barro and Sala-i-Martin 1991, 1992) could also be expressed with the help of these phase diagrams. In this case the curve would be a straight line with positive or negative slope. A negative slope would correspond to stable dynamics associated with a tendency of all countries to converge to the same steady-state level of per capita income. This convergence would be caused by relatively poor countries growing faster than relatively rich countries and the richest countries actually shrinking towards the steady state. By contrast, a positive slope would correspond to unstable dynamics and divergence of per capita income levels. Empirical investigations in the literature routinely find divergence across countries according to this concept (see Sala-i-Martin 1996).
In the following two sections we provide parametric and nonparametric estimates of the dynamics of per capita income. For the parametric estimates a third-order polynomial specification is used that nests the functions leading to both variants of the phase diagram shown above as well as the linear functional form that is used in the assessment of $\beta$-convergence. The resulting curve fit is then compared to a nonparametric estimate to validate the appropriateness of the third-order polynomial functional form.

### 3 Parametric and Nonparametric Estimates

Because of the uncertain shape of the function $A(\cdot)$ leading to the uncertain shape of the function $T(\cdot)$ we follow two different routes of estimation. The first consists of fitting the above-mentioned third-order polynomial specification of $T(\cdot)$ by ordinary regression methods. The second consists of not imposing any assumptions about the functional form of $T(\cdot)$ and fitting a purely nonparametric regression by kernel methods instead.

To estimate the conditional mean of a dependent variable, ordinary regression solves the least-squares problem

$$\min_{\beta \in \mathbb{R}^3} \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i - \beta_2 x_i^2 - \beta_3 x_i^3)^2,$$

where in the following the dependent variable $y$ is the average growth rate of per capita income over the 5-year intervals and $x$ is the logarithm of the level of per capita income at the start of the respective 5-year interval.

As explained e.g. in Wand and Jones (1995), the local polynomial kernel estimator of order 3 solves

$$\min_{\theta \in \mathbb{R}^3} \sum_{i=1}^{n} (y_i - \theta_0 - \theta_1(x_i - x_0) - \theta_2(x_i - x_0)^2 - \theta_3(x_i - x_0)^3)^2 \cdot K((x_i - x_0)/h)$$

and estimates the ordinate of the regression function at a selected point $x_0$ by $\hat{f}(x_0) = \hat{\theta}_0$. The kernel function $K(\cdot)$ serves to assign higher weights to those observations with $x$-values closer
to $x_0$. Here a Gaussian kernel (the standard normal density) is used as the kernel function and the bandwidth parameter $h$ is selected according to the direct plug-in method of Ruppert et al. (1995). Doubling this bandwidth is used to induce oversmoothing for obtaining a smoother final result. It is important to notice that the order 3 of the local polynomial estimator is in no way related to the order 3 of the polynomial that is fitted by least squares. Instead, the local polynomial estimator is entirely nonparametric in not imposing any a priori functional form of the regression relationship. The order 3 of the local polynomial estimator is chosen to reduce biases, especially at the boundaries of the range of the $x$-values as suggested by Wand and Jones (1995, pp. 126ff.).

Figure 2 shows the results of the two function estimates superimposed on a scatter-plot of the observations. The estimates rely on data from the Penn World Table 6.2 spanning the period
1960-2003 for the 96 countries used here. The data series used is real GDP per capita, deflated by a chain index. All data points are specified in natural logarithms. See Summers and Heston (1988) and Aten et al. (2006) for a detailed description of the data set.

Actually used for the analysis are values in 5-year intervals to gain a larger number of observations and to weaken the effects of short-run fluctuations. (Note that in the figure the axis label $\Delta \ln y_{t+5}$ is understood as $\ln y_{t+5} - \ln y_t$.) The first reason is particularly important for the nonparametric regression fits and for the quantile regression fits later on. Using a pure cross-section of countries produced rather wide confidence intervals, thereby preventing any reasonable interpretation of the results. The second reason is also highlighted by Fiaschi and Lavezzi (2007), who on the one hand emphasize that relying on intervals over several years allows to circumvent problems arising from autocorrelated shocks as well as measurement errors and on the other hand point to the robustness of the results to the length of the interval. We also abstain from applying the transformations common in panel data analysis such as the within-transform since these would inevitably destroy the correspondence of the estimated equation and its theoretical counterpart. This choice also follows Fiaschi and Levezzi (2007), Kalaitzidakis et al. (2001), and Liu and Stengos (1999).

In the figure the solid line shows the fit of a third-order polynomial, fitted by least squares. It is surrounded by two dotted lines, representing 95% confidence bounds for the least squares fit. The fitted function has the shape of a wave that would be compatible with a poverty-trap pattern if it crosses the zero line which evidently is not the case here. This finding is confirmed by the nonparametric fit from the local polynomial kernel smoother shown as the dashed line. Quite assuring for the third-order polynomial specification is that the dashed line tracks the solid line reasonably closely. Main exception is the right margin where the nonparametric fit is locally attracted by some larger growth rates of high-income observations. Since the fitted curve totally lie above the horizontal axis, this leads to the conclusion that all countries tend to grow irrespective of their income levels.

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2 The two countries Equatorial Guinea and Ghana have been excluded because they appear to be obvious outliers in the scatter-plots. Their exclusion has almost no effect on the fitted curves but lets the cloud of points appear to be more compact.
Figure 3 shows the results of the same analysis with data on real GDP per worker instead of real GDP per capita, again taken from the Penn World Table 6.2. The estimated curve here again lies entirely above the horizontal axis and now appears to be almost flat. The nonparametric fit is also again tracking the third-order polynomial specification reasonably well. Shape and position of the fitted curves here again indicate the absence of $\beta$-convergence in its absolute form and the absence of poverty traps of any kind.

We must be careful with this conclusion, however, since the estimated curves (parametric as well as nonparametric) represent estimates of the mean of the growth rate distribution, conditioned only on the lagged level of per-capita income. The effects of the introduction of further conditioning variables on the finding of nonlinearity in the growth process are explored in the semiparametric analyses of Kalaitzidakis et al. (2001) and of Liu and Stengos (1999). In both
cases the basic curvature of the relation of income growth and income levels appears to be robust.

Since the above results apply to the conditional mean of the growth rate, the only assertion we can make with confidence is that the poverty-trap pattern of dynamics does not apply in the mean or for a representative (average) country. Thus, the above analysis is also vulnerable to the critique by Quah (1996, 1997) for the case of the regression estimates to establish $\beta$-convergence. Quah strongly suggests to pay much more attention to the dynamics of the whole distribution of per capita income instead of just its mean. This is exceptionally clearly stated by Durlauf and Quah (1999, p. 294) in saying that “explaining distribution dynamics’ needs to go beyond representative-economy analysis”.

In the next section we take up the distribution-dynamics perspective using the method of quantile regression which estimates the effect of the changes of conditioning variables on the position of a particular quantile of the distribution of the dependent variable, i.e. the growth rate of per capita income over five-year intervals. Quantile regression has the potential to uncover different shapes of a regression function for different quantiles of the dependent variable. This allows to implement the regression analysis for different quantiles and to obtain (potentially) different shapes of the regression function by which the poverty-trap dynamics may be characterized more completely. Thus, it is possible not only to discern whether different dynamics are at work for countries with high or low levels of income but simultaneously distinguishing countries with high or low rates of income growth. In addition to this, the quantile regression approach has certain features of a semiparametric method such as the improved robustness with respect to outliers of the dependent variable.

4 Quantile Regression Estimates

Quantile regression has been introduced by Koenker and Bassett (1978). The recent book by Koenker (2005) gives a comprehensive account of the basic approach and the different paths further developments of quantile regression have taken. In parallel to the previous section we consider here ordinary quantile regression, which is the quantile analog to ordinary linear
Ordinary quantile regression solves the problem

$$\min_{\beta \in \mathbb{R}^3} \sum_{i=1}^{n} \rho_\tau (y_i - \beta_0 - \beta_1 x_i - \beta_2 x_i^2 - \beta_3 x_i^3),$$

where the "check function" $\rho_\tau (u) = u \cdot (\tau - I(u < 0))$ controls which quantile $\tau$ of the dependent variable is considered ($I(\cdot)$ denotes the usual indicator function which is equal to one if $u < 0$ and zero otherwise). By that the resulting parameter vector becomes specific to the particular quantile considered and consequently the shape of the estimated regression function becomes also specific to that quantile. The dependent variable $y$ and the explanatory variable $x$ are defined as above.

A nonparametric variant of quantile regression using smoothing splines as suggested by Koenker et al. (1994) solves the problem

$$\min_{g \in \mathcal{G}} \sum_{i=1}^{n} \rho_\tau (y_i - g(x_i)) + \lambda \int |g''(x)| dx.$$

Searched here is a function $g(\cdot)$ for a specific quantile $\tau$. This function is specified as a spline. It is supposed to fit the data as good as possible, simultaneously respecting the roughness penalty (the total variation norm) in the second summand. The importance of this term and thus of the degree of smoothing is controlled by the parameter $\lambda$. After some experimentation this smoothing parameter is set equal to two. The result is a regression function estimate that is specific to the particular quantile considered and does not rely on any a priori assumptions regarding its functional form.

Figure 4 presents the results for real GDP per capita with the solid line showing the third-order polynomial, fitted by quantile regression for the quintiles of the growth rate distribution.\(^3\) The dotted lines indicate bootstrapped 95% confidence intervals from 10000

\(^3\) For the case of five-year growth rates of real GDP per capita the four quintiles are at −0.01, 0.07, 0.13 and 0.2, respectively.
bootstrap repetitions of the quantile fit. It is also apparent that the solid line is again closely tracked by the dashed line representing the nonparametric quantile fit.

Considering first the upper quintiles (with $\tau$ equal to 0.6 and 0.8) in the second row of the figure the situation is much the same as for the mean regressions in the previous section. For the upper quintiles no poverty trap can be detected. In the case $\tau = 0.4$ we observe that the fitted curve (or better the range between the confidence bounds) almost touches the horizontal axis at a log per capita income level of about 7. This could be an unstable fixed point, a so-called "shunt" in the dynamic analysis literature, which is simply passed by the dynamics coming from the left and going to the right. At higher per capita income levels (with a log above 10) the fitted curve may cross the horizontal axis leading to a stable fixed point there. Altogether, there appears to be no poverty trap in this case also.

Figure 4
Quantile Regression Fits for Real GDP per Capita

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Quite different is the picture for the lowest quintile ($\tau = 0.2$). There, the fitted curve and its confidence interval is below the horizontal axis at log per capita income levels of around 7 and above the horizontal axis at levels between 9 and 10. At levels above 10 the fitted curve does not cross the horizontal axis but the confidence interval widens and its lower bound becomes negative. Therefore, the fitted curve not only has the right shape but also leads to a dynamic pattern with a fixed point at a relatively low level of per capital income. This fixed point is stable and it is also statistically significant in the sense that the confidence interval around the fitted curve is completely below and above the horizontal axis for the theoretically relevant levels of per capita income.

If there is a second stable fixed point at relatively high income levels depends on whether one relies on the course of the confidence interval or on the course of the solid line representing the fitted curve. In the first case we would conclude in favor of a dynamic pattern according to variant 1 of the phase diagram discussed above. Then the second fixed point would be considered as stable. In the second case the dynamic pattern would resemble variant 2 of the above phase diagram with sustained per capita income growth once the critical threshold has been surmounted. In the light of the results for real GDP per worker discussed below, the second case appears to be more plausible compared to the first one.

Looking at specific countries we find many developing countries mainly in sub-Saharan Africa being in the poverty trap by the year 2000. Examples for countries in the vicinity of the threshold per-capita income level of $\exp(8.5) \approx 4900$ are El Salvador, Paraguay or Ukraine. Above a per capita income level of $\exp(10) \approx 22000$ by the year 2000 we find many OECD and oil-exporting countries (since we use the Penn World Table 6.2 all per capita income levels are expressed in international dollars of the year 2000). Despite the fundamentally different approaches, the results reported here are roughly in accord with findings of Semmler and Ofori (2007), who state that "about 14% of countries will converge to a steady state with an income only a quarter of the average" (p. 21).
Analogous to the previous figure, figure 5 shows the results using real GDP per worker instead of real GDP per capita.\textsuperscript{4} Again it is apparent that only in the case of the lowest quintile ($\tau = 0.2$) the poverty-trap phenomenon appears. Instead of a wave-shaped pattern, the fitted curve is clearly u-shaped with a stable fixed point at low levels of income per worker and an unstable fixed point at high levels of income per worker. Between both fixed points the curve as well as the confidence band is entirely below the horizontal axis, thus also statistically supporting the conclusion of the possibility of a poverty trap for the lowest growth quintile. Given the threshold level of a per-capita income level of slightly above $\exp(10) \approx 22000$ it seems to be rather unlikely that countries in the lowest growth quintile will be able to manage the transition to the sustained growth regime. Thus even countries with quite high per-capita income levels are in danger to be driven back to lower income levels if they are not able to

\textsuperscript{4} For the case of five-year growth rates of real GDP per worker the four quintiles are at −0.02, 0.06, 0.12 and 0.19, respectively, slightly lower than for real GDP per capita.
pass the critical threshold level. For the other quintiles no indication for the poverty-trap phenomenon could the gained from the data.

5 Conclusion

With the application of quantile regression in the present paper more light could be shed on the poverty-trap phenomenon. The occurrence of this phenomenon in international growth data was already explored in the recent literature on nonlinear growth by Fiaschi and Lavezzi (2003, 2007), Liu and Stengos (1999) and Kalaitzidakis et al. (2001) using semi- and nonparametric regression techniques. Since these results were not entirely clear-cut and since these regression techniques apply to the mean of the growth rate distribution, the present paper contributes to the literature by a separate analysis of different quantiles of the growth rate distribution. This is much in the spirit of the distribution-dynamics approach advocated by Quah (1996, 1997), but technically implemented there using tools different from the quantile regression approach. Exactly this quantile regression approach is applied in the present paper in both parametric and nonparametric variants.

The essence of these estimates is that the phenomenon of a poverty-trap (i.e. the danger of being trapped in an equilibrium associated with a sustained low level of per capita income) appears to be limited to countries with low levels of per capita income appearing simultaneously with growth rates around and below the lowest quintile of the growth rate distribution. Thus being disadvantaged both in terms of per capita income levels and in terms of per capita income growth appears to be mutually reinforcing and leads to being caught in a stable low-income equilibrium. Conversely, for all other countries with higher growth rates or higher per capita income levels a poverty trap seems not to be a real danger. The estimates reported in this paper provide direct evidence on this phenomenon and the underlying mechanism. They support the opinion that models with multiple equilibria leading to poverty traps are indeed an essential and empirically relevant part of the growth economists tool kit.
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